Fusing Dependent Decisions for Hypothesis Testing With Heterogeneous Sensors

Satish G. Iyengar, Ruixin Niu, *Senior Member, IEEE*, and Pramod K. Varshney, *Fellow, IEEE*

***Abstract*—In this paper, we consider a binary decentralized detection problem where the local sensor observations are quantized before their transmission to the fusion center. Sensor observations, and hence their quantized versions, may be heterogeneous as well as statistically dependent. A composite binary hypothesis testing problem is formulated, and a copula-based generalized likelihood ratio test (GLRT) based fusion rule is derived given that the local sensors are uniform multilevel quantizers. An alternative computationally efficient fusion rule is also designed which involves injecting a deliberate random disturbance to the local sensor decisions before fusion. Although the introduction of external noise causes a reduction in the received signal-to-noise ratio (SNR), it is shown that the proposed approach can result in a detection performance comparable to the GLRT detector without external noise, especially when the number of quantization levels is large.**

***Index Terms*—Copula theory, hypothesis testing, multimodal signals, multisensor fusion, quantization, statistical dependence, stochastic resonance.**

I. Introduction

Decentralized detection has long been an active and important research area [1]. One of the earliest applications to have motivated research in decentralized detection was distributed radar where it was essential to compress data at each (local) radar before relaying it to a fusion center (see [2] and references therein). More recently, decentralized detection has found applications in sensor networks. Recent technological advances have enabled the deployment of multiple low cost sensors to monitor a region of interest (ROI) for reliable detection, estimation and/or tracking of events. Each sensor is usually programmed to send only quantized versions of its measurements to a remotely located fusion center due to communication bandwidth and power constraints. The fusion center then consolidates receptions from all the sensors to make a global inference.

The design of a distributed detection system involves designing the local and fusion center decision rules under different criteria and constraints [1], [3], [4]. Under the assumption that local observations are conditionally independent given the hypothesis, and the fusion center receives the local sensor outputs without any loss, the optimality of the LRT for local sensor decision rules under the Bayesian criterion and the Neyman–Pearson criterion have been proved in [5] and [6]. However, the problem becomes highly complex when conditional independence assumption does not hold. The LR-based decision rules at the local sensors may no longer result in an optimal system design [7], [8]. It has also been shown that distributed detection with dependent observations is an NP-complete problem; it cannot be solved using a polynomial time algorithm [9], [10]. The problem is usually simplified by constraining the local sensors to be binary quantizers. In [11], Drakopoulos and Lee have derived a rule for fusing correlated decisions under the assumption that the correlation coefficients between the sensor decisions are known and local sensor thresholds generating the correlated decisions are given. Kam *et al.* [12], employed another approach, namely, the Bahadur–Lazarsfeld series expansion of probability density functions (pdfs) to derive the optimum fusion rule for correlated local decisions. It was, however, assumed that the joint distribution of sensor observations was completely known. In this paper, we consider the scenario in which the dependence structure and hence the joint distribution between the sensor observations may be unknown. Such problems are typical of sensor networks that consist of heterogeneous sensors, i.e., sensors with disparate sensing modalities. For example, it is not immediately clear how one could model the complex relationship between observations of an audio and a video sensor monitoring a common ROI.

The problem of binary hypothesis testing with heterogeneous sensors has been considered in our earlier work [13], where we developed a parametric framework using the statistical theory of copulas. While designing the copula based fusion rule in [13], it was, however, assumed that the fusion center has access to the exact real-valued (analog) version of the locally processed data. In many cases such as in WSNs discussed above, there could be limitations on both the transmission power and the bandwidth available for sensor-to-fusion center communication. It may, thus, be necessary to quantize the information at each sensor before its transmission to the fusion center. Our goal, in this paper, is to design a decision fusion rule based on copula theory, for combining quantized heterogeneous information, thereby significantly extending the formulation and results in [13]. We note here that the application of copula theory for fusing correlated decisions has been recently considered in [14]. The local sensors were binary quantizers, and it was assumed that the *true* copula function generating the data under each hypothesis is known *a priori*, except for some parameters. In this work, we relax this

Manuscript received October 16, 2011; revised March 20, 2012; accepted May 02, 2012. Date of publication June 01, 2012; date of current version August 07, 2012. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Roberto Lopez-Valcarce. This work was supported by the Army Research Office, and was accomplished under the Cooperative Agreement No. W911NF-09-1-0244. It was also supported by the AFOSR under Grant FA-9550-09-1-0064.

S. G. Iyengar and P. K. Varshney are with the Department of Electrical Engineering and Computer Science, Syracuse University, Syracuse, NY 13244 USA (e-mail: iyengar@ge.com; siyengar@syr.edu; varshney@syr.edu).

R. Niu is with the Department of Electrical and Computer Engineering, Virginia Commonwealth University, Richmond, VA 23284 USA (e-mail: rniu@vcu.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TSP.2012.2202113

assumption and consider the case when the copula function used to model the dependence structure between the variables may be "misspecified," i.e., the chosen copula density may not accurately characterize intersensor dependence. The formulation is also extended to include multibit quantizers at the local sensors.

As will be evident later, one of the main limitations of the copula-based generalized likelihood ratio test (GLRT) for fusing discrete decisions is the considerable increase in computational complexity as the number of sensors and/or quantization levels increases. For example, a system with N sensors each with an M-level quantizer requires the computation of N-dimensional integrals, and optimization over an $\frac{N(N-1)}{2}$-dimensional space for maximum likelihood (ML) estimation of parameters associated with elliptical copulas such as the Gaussian and t-copula functions [15]. This issue of computational complexity is also addressed in this paper, and an alternative computationally efficient fusion rule is proposed that involves deliberately adding external noise to the quantized observations before fusion. We call this noise, the low-pass filter (LPF) noise for reasons that will become clear later. The approach completely eliminates the necessity to compute the multidimensional integrals and greatly simplifies the fusion rule. However, the reduced complexity comes at the cost of the decreased signal-to-noise ratio (SNR) at the fusion center. Thus, the key to the success of this approach is a "good" design of the LPF noise, i.e., we need to derive the form of the LPF-noise pdf that would introduce minimal distortion. We present an approach based on Widrow's additive quantization noise model. Our approach is similar to Gustafsson and Karlsonn [16] who have considered the problem of estimating a deterministic parameter in noise using quantized observations. However, unlike [16], where the authors propose to inject the artificial dither noise before quantization, we add the deliberate disturbance post quantization, and at the fusion center. As we show later, the addition of noise after quantization is equivalent to low pass filtering in the characteristic function (CF) domain, unlike dithering which amounts to anti-alias filtering [16], [17].

The paper is organized as follows. The problem is formulated in Section II, and a copula-based rule for fusing dependent local sensor decisions is derived in Sections III and IV. Section V addresses the issue of computational complexity associated with the fusion rule derived in the previous sections. An alternative computationally efficient fusion rule based on Widrow's statistical theory of quantization is proposed here. An illustrative example is presented in Section VI to elucidate the theory presented in the previous sections. In Section VII, we identify a class of problems for which the detector threshold can be determined to achieve a desired false alarm rate. We summarize our paper and provide some concluding remarks in Section VIII.

II. Problem Formulation

The problem of signal detection is formulated as a binary hypothesis test where the hypothesis H_1 indicates the presence of a signal, while H_0 indicates its absence. A total of N sensors are used to collect observations, Z_n, for $n = 1, \ldots, N$. Observations at each sensor n are independent and identically distributed (i.i.d.) over time with pdfs $f_n(z_n; \psi_n)$ and $g_n(z_n; \lambda_n)$ under H_1 and H_0 respectively, where ψ_n and λ_n are distributional parameters. We assume that these marginal pdfs are well-specified under both hypotheses (see Definition 1 below).

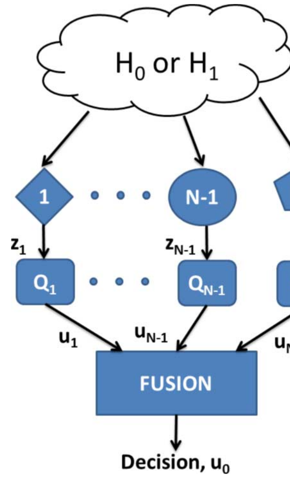


Fig. 1. Distributed heterogeneous sensor network: A parallel architecture.

Definition 1—Well-Specified Model (White 1994): A parametric model $\{f(x;\Theta)\}$ is *well-specified* for a random variable X if there exists a unique $\theta' \in \Theta$ such that $f(x;\theta') \in \{f(x;\Theta)\}$ corresponds to the true density of X. Otherwise, $\{f(x;\Theta)\}$ is said to be *misspecified* for X.

However, no knowledge is assumed regarding the dependence structure between the heterogeneous data streams. We approximate this dependence using copula functions (see Section II-A below). Sensor observations are further passed through uniform multilevel quantizers (see Fig. 1) before their transmission to a remotely located fusion center. The input-output transfer function of the quantizer at each sensor is shown in Fig. 2. Thus, the quantizer output, during any time interval $1 \leq l \leq L$, can be given as

$$u_{nl} = \mathcal{Q}_m(z_{nl}) = \begin{cases} -m_n q_n - \frac{q_n}{2}, & z_{nl} < -m_n q_n, \\ q_n \lfloor \frac{z_{nl}}{q_n} \rfloor + \frac{q_n}{2}, & -m_n q_n < z_{nl} \leq m_n q_n, \\ m_n q_n + \frac{q_n}{2}, & z_{nl} \geq m_n q_n \end{cases} \quad (1)$$

where q_n and $2(m_n + 1)$ correspond to the quantizer step size and the number of quantization levels, respectively, at sensor n. Further, $\lfloor x \rfloor$ stands for the floor operation that denotes an integer smaller than or equal to x. The quantized value at sensor n can be represented with an integer $i_n = -m_n - 1, -m_n, \ldots, m_n + 1$. In this paper, we do not consider quantizer saturation errors. That is, we assume that the dynamic range of the (analog) signal input to the quantizer is well within the lower and upper limits of the quantizer.

Observations thus received at the fusion center are used to estimate the unknown model parameters, and a GLRT-based fusion rule is employed for global decision making. In addition to estimating the model parameters, the selection of copula densities is also embedded in the GLRT formulation and is thus performed in real time. Sensor observations and hence their quantized versions are assumed to be i.i.d. in time, and, our focus, in this paper, is on designing a fusion rule that could exploit the spatial dependence between sensor decisions for improved detection performance.

Next, we briefly discuss the use of copula theory to approximate joint density functions.

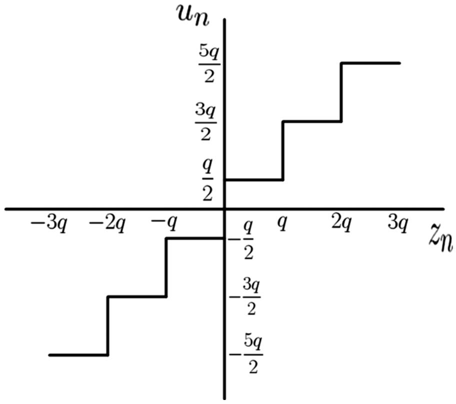


Fig. 2. Input–output transfer function of a uniform scalar quantizer.

A. Joint pdf Approximation Using Copula Theory

We begin with the definition of a copula function.

Definition 2: A function $C : [0,1]^N \rightarrow [0,1]$ is an N-dimensional copula if C is a joint cumulative distribution function (cdf) of an N-dimensional random vector on the unit cube $[0,1]^N$ with uniform marginals [15], [19], [20].

The following theorem by Sklar is central to the statistical theory of copulas.

Theorem 1 (Sklar's Theorem): Let F be an N-dimensional cdf with continuous marginal cdf's $F_1, F_2, \dots, F_N$. Then there exists a unique copula C such that for all $z_1, z_2, \dots, z_n$ in $[-\infty, \infty]$,

$$F(z_1, z_2, \dots, z_N) = C(F_1(z_1), F_2(z_2), \dots, F_N(z_N)). \quad (2)$$

Note that the copula function $C(u_1, u_2, \dots, u_N)$ is itself a cdf with uniform marginals as $U_n = F_n(Z_n) \sim \mathcal{U}(0,1)$ (by probability integral transform). The joint density can now be obtained by taking the N^{th} order derivative of (2),

$$\begin{aligned} f(\mathbf{z}) &= \frac{\partial^N}{\partial z_1 \dots \partial z_N} C(F_1(z_1), \dots, F_N(z_N)) \\ &= \underbrace{\left(\prod_{n=1}^{N} f_n(z_n) \right)}_{f_p(\mathbf{z})} c(F_1(z_1), \dots, F_N(z_N)). \end{aligned} \quad (3)$$

Thus, in (3), the copula density, $c(\cdot)$, *weights* the product density, $f_p(\mathbf{z})$, *appropriately* to incorporate dependence between the random variables $\{Z_n\}_{n=1}^N$.

Theorem 1 also admits the following converse, especially useful in practice when the true distribution F (and hence the true copula C) is unknown. It allows one to construct a statistical model by considering the univariate behavior of the underlying marginals and subsequently fitting the desired dependence structure specified by some copula, say, K.

Theorem 2: If $F_1, F_2, \dots, F_N$ are univariate marginal cdfs and if K is an N dimensional copula, then the function $\Xi : \mathbb{R}^N \rightarrow [0,1]$

$$\Xi(z_1, \dots, z_N) = K(F_1(z_1), \dots, F_N(z_N)) \quad (4)$$

is a valid N-variate cdf with marginals $F_1, F_2, \dots, F_N$. A copula based parametric model can be derived by taking the N^{th} order derivative of (4) to obtain

$$\begin{aligned} \hat{f}(\mathbf{z}) &= f_p(\mathbf{z}) k(F_1(z_1), \dots, F_N(z_N)) \\ &= f_k(\mathbf{z}). \end{aligned} \quad (5)$$

Some of the commonly used copula functions include the Gaussian, Student's t and those belonging to the Archimedean family [13].

It is evident that model mismatch errors are introduced when $k(\cdot) \neq c(\cdot)$; i.e., the selected copula does not represent the true dependence structure. This leads to suboptimal performance. An important question then is, *how does one choose $k(\cdot)$ from a finite set (say $\mathcal{A}_k$) of copula densities*? As discussed earlier, the selection of copula densities is embedded in the GLRT formulation in this paper (see (14)). It may also be required to estimate the parameters, ψ_d, of the chosen copula function from the acquired data. These parameters control the shape of the copula function and can be estimated by exploiting their relations to other nonparametric measures of association such as Kendall's τ or Spearman's ρ [15]. In this paper, we use a maximum-likelihood (ML) based approach known as the method of inference functions for margins (IFM) [21] to estimate the copula dependence parameters.

III. (Misspecified) GLRT-Based Fusion of Soft Decisions

In the following, we consider a two-sensor network for simplicity.

Under hypothesis H_1, the probability that the data $R_l = (u_{1l}, u_{2l})$ received at the fusion center at the time instant l takes a specific value $\left(i_1 q_1 + \frac{q_1}{2}, i_2 q_2 + \frac{q_2}{2}\right)$ is

$$P_{i_1,i_2} = \int_{i_1 q_1}^{(i_1+1)q_1} \int_{i_2 q_2}^{(i_2+1)q_2} f(z_1, z_2) dz_2 dz_1, \quad (6)$$

where $f(z_1, z_2)$ is the true but unknown joint pdf of unquantized sensor observations under H_1. Now, approximating the dependence structure using a copula density $k_1 \{F_1(z_1; \psi_1), F_2(z_2; \psi_2); \psi_d\}$ contained in some set $\mathcal{A}_k$ of valid copula densities, we have

$$\begin{aligned} &\hat{P}_{i_1,i_2}(\psi) \\ &\quad = \int_{i_1 q_1}^{(i_1+1)q_1} \int_{i_2 q_2}^{(i_2+1)q_2} \hat{f}(z_1, z_2; \psi) dz_2 dz_1 \\ &\quad = \int_{i_1 q_1}^{(i_1+1)q_1} \int_{i_2 q_2}^{(i_2+1)q_2} f_1(z_1; \psi_1) f_2(z_2; \psi_2) \\ &\qquad \times k_1 \left(F_1(z_1; \psi_1), F_2(z_2; \psi_2); \psi_d\right) dz_2 dz_1 \\ &\quad = K_1 \left\{F_1\left((i_1+1)q_1; \psi_1\right), F_2\left((i_2+1)q_2; \psi_2\right); \psi_d\right\} \\ &\qquad - K_1 \left\{F_1\left(i_1 q_1; \psi_1\right), F_2\left(i_2 q_2; \psi_2\right); \psi_d\right\}, \end{aligned} \quad (7)$$

where $\psi = (\psi_1, \psi_2, \psi_d)^T \in \Psi \subset \mathbb{R}^a$ is the a-dimensional unknown parameter vector that will be estimated from the received data, $K_1\{\cdot\}$ is the copula cdf and $F_n(\cdot)$ is the cdf of Z_n under hypothesis H_1. The dependence of $\hat{P}_{i_1,i_2}(\psi)$ on $K_1\{\cdot\}$ is not made explicit for notational convenience.

The likelihood function of the data R_l under hypothesis H_1 can now be written as

$$\hat{P}(R_l;\psi,H_1) = \prod_{i_1}\prod_{i_2}\left[\hat{P}_{i_1,i_2}(\psi)\right]^{\delta\left(u_{1l}-i_1q_1-\frac{q_1}{2},u_{2l}-i_2q_2-\frac{q_2}{2}\right)} \quad (8)$$

where $\delta(\cdot)$ is the two-dimensional Kronecker-delta function defined as

$$\delta(x,y)=\begin{cases}1, & x=y=0\\ 0, & \text{otherwise.}\end{cases} \quad (9)$$

The log-likelihood function of R_l is, therefore

$$\log\hat{P}(R_l;\psi,H_1)=\sum_{i_1}\sum_{i_2}\delta\left(u_{1l}-i_1q_1-\frac{q_1}{2},u_{2l}-i_2q_2-\frac{q_2}{2}\right)\log\hat{P}_{i_1,i_2}(\psi). \quad (10)$$

Similarly, the likelihood function of R_l under H_0, when a copula density $k_0\left(G_1(z_1;\lambda_1),G_2(z_2;\lambda_2);\lambda_d\right)\in\mathcal{A}_k$ is used to approximate the joint distribution under H_0, can be derived as,

$$\log\hat{P}(R_l;\lambda,H_0)=\sum_{i_1}\sum_{i_2}\delta\left(u_{1l}-i_1q_1-\frac{q_1}{2},u_{2l}-i_2q_2-\frac{q_2}{2}\right)\log\hat{Q}_{i_1,i_2}(\lambda) \quad (11)$$

where

$$\begin{aligned}\hat{Q}_{i_1,i_2}(\lambda)=&K_0\left\{G_1\left((i_1+1)q_1;\lambda_1\right),\right.\\&\left.G_2\left((i_2+1)q_2;\lambda_2\right);\lambda_d\right\}\\&-K_0\left\{G_1\left(i_1q_1;\lambda_1\right),G_2\left(i_2q_2;\lambda_2\right);\lambda_d\right\}. \quad (12)\end{aligned}$$

$\lambda=(\lambda_1,\lambda_2,\lambda_d)^T\in\Lambda\subset\mathbb{R}^b$, is the b-dimensional unknown parameter vector, $K_0\{\cdot\}$ is the copula cdf and $G_n(\cdot)$ is the cdf of Z_n under hypothesis H_0. With (10) and (11), it is straightforward to derive the test to be employed at the fusion center

$$\mathcal{T}_k(\mathbf{u}_1,\mathbf{u}_2)\triangleq\log\frac{\max\limits_{k_1(\cdot)\in\mathcal{A}_k,\Psi}\prod_l\hat{P}(R_l;\psi,H_1)}{\max\limits_{k_0(\cdot)\in\mathcal{A}_k,\Lambda}\prod_l\hat{P}(R_l;\lambda,H_0)}\underset{H_0}{\overset{H_1}{\gtrless}}\eta \quad (13)$$

which results in

$$\sum_l\sum_{i_1}\sum_{i_2}\delta\left(u_{1l}-i_1q_1-\frac{q_1}{2},u_{2l}-i_2q_2-\frac{q_2}{2}\right)\times\log\frac{\hat{P}^*_{i_1,i_2}(\hat{\psi})}{\hat{Q}^*_{i_1,i_2}(\hat{\lambda})}\underset{H_0}{\overset{H_1}{\gtrless}}\eta \quad (14)$$

where $\hat{P}^*_{i_1,i_2}(\hat{\psi})$ and $\hat{Q}^*_{i_1,i_2}(\hat{\lambda})$ correspond to the copula functions $K_1^*(\cdot)$ and $K_0^*(\cdot)$, respectively, which maximize the terms on the left hand side of (13), and l is the time index. Thus, the maximization in (13) is over the copula densities belonging to a set $\mathcal{A}_k$ of valid copula densities as well as the unknown marginal and copula dependence parameters. Unlike the classical composite hypothesis testing formulation which would have required the knowledge of the true copula densities with possibly unknown parameters, we allow for the case when the set $\mathcal{A}_k$ may not be inclusive of the true models $c_1(\cdot)$ and/or $c_0(\cdot)$. Thus, the copula functions, $K_1^*(\cdot)$ and $K_0^*(\cdot)$, chosen after maximization may still be misspecified. We, therefore, call the test a misspecified GLRT (mGLRT).

IV. Extension to N (>2) Sensors

The copula based fusion rule designed for a two sensor network in the previous section can be easily extended to larger sensor networks. Similar to (14), the fusion rule for N sensors can be derived as

$$\sum_l\sum_{i_1}\cdots\sum_{i_N}\delta\left(u_{1l}-i_1q_1-\frac{q_1}{2},\ldots,u_{Nl}-i_Nq_N-\frac{q_N}{2}\right)\times\log\frac{\hat{P}^*_{i_1,\ldots,i_N}(\hat{\psi})}{\hat{Q}^*_{i_1,\ldots,i_N}(\hat{\lambda})}\underset{H_0}{\overset{H_1}{\gtrless}}\eta \quad (15)$$

where,

$$\begin{aligned}\hat{P}^*_{i_1,\ldots,i_N}(\psi)=&\int_{i_1q_1}^{(i_1+1)q_1}\cdots\int_{i_Nq_N}^{(i_N+1)q_N}\underbrace{\hat{f}(z_1,\ldots,z_N;\psi)}_{\text{copula}-\text{based estimate}}\,dz_N\cdots dz_1 \quad (16)\\=&K_1^*\left\{F_1\left((i_1+1)q_1;\psi_1\right),\ldots,F_N\left((i_N+1)q_N;\psi_N\right);\psi_d\right\}\\&-K_1^*\left\{F_1\left(i_1q_1;\psi_1\right),\ldots,F_N\left(i_Nq_N;\psi_N\right);\psi_d\right\} \quad (17)\end{aligned}$$

and,

$$\begin{aligned}\hat{Q}^*_{i_1,\ldots,i_N}(\lambda)=&\int_{i_1q_1}^{(i_1+1)q_1}\cdots\int_{i_Nq_N}^{(i_N+1)q_N}\underbrace{\hat{g}(z_1,\ldots,z_N;\lambda)}_{\text{copula}-\text{based estimate}}\,dz_N\cdots dz_1 \quad (18)\\=&K_0^*\left\{G_1\left((i_1+1)q_1;\lambda_1\right),\ldots,G_N\left((i_N+1)q_N;\lambda_N\right);\lambda_d\right\}\\&-K_0^*\left\{G_1\left(i_1q_1;\lambda_1\right),\ldots,G_N\left(i_Nq_N;\lambda_N\right);\lambda_d\right\}. \quad (19)\end{aligned}$$

Thus, the fusion rule involves evaluating N-dimensional integrals in real-time where N is the number of sensors, i.e., the computational complexity is exponential in the number of sensors. This is in addition to the optimization over multiple dimensions to obtain ML estimates of the unknown parameters. Application of mGLRT is, therefore, highly prohibitive as the number of sensors increases due to the increased computational complexity. We derive an alternative computationally efficient test in the next section.

V. A Computationally Efficient Fusion Rule

In this section, we propose a computationally efficient approach that involves deliberately injecting noise to the quantized observations before fusion (see Fig. 3). While noise is generally perceived as an unwanted signal, interestingly, several studies have shown that the addition of *controlled noise* could in fact be beneficial in some cases. For example, dithering, the process of adding noise to the signal before quantization has been shown to improve signal quality and mitigate the artifacts

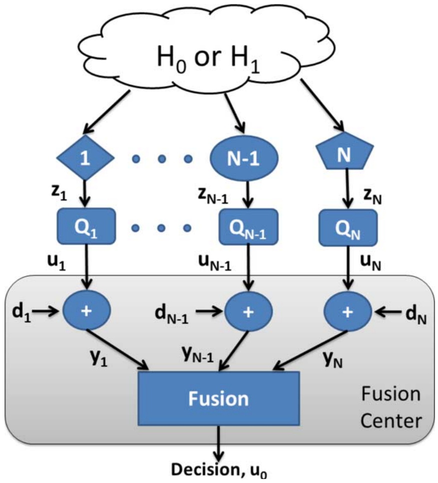


Fig. 3. A *controlled* noise d_n is added at the output of each sensor n. The approach greatly simplifies the fusion rule by avoiding the need to compute multidimensional integrals.

introduced due to quantization [22]–[24]. Also, it has been observed by many researchers that some types of signals get amplified by a nonlinear system when noise is added to the input signal (see [25], and references therein). This phenomenon is popularly known as stochastic resonance (SR). Here, we use this approach of adding external noise to reduce computational complexity rather than to enhance the signal-to-noise ratio (SNR). Our approach is based on Widrow's quantization theory which we review next.

A. Widrow's Statistical Theory of Quantization: A Review

The statistical theory of quantization was developed by Widrow and co-workers [17], [26], [27]. They interpreted quantization of a random variable as sampling of its pdf, and showed that the pdf of the quantized signal is the convolution of the input signal pdf with a rectangular pulse function followed by conventional sampling. Thus, the pdf of the quantizer output, u_{nl}, at sensor n and at any time instant, l, can be given as

$$p_{U_n}(z) = \left(p_{W_n}(z) \star p_{Z_n}(z)\right) \cdot c_{\delta'_n}(z) \tag{20}$$

where $p_{Z_n}(z)$ is the pdf of the random variable at the input Z_n, $p_{W_n}(z)$ denotes the rectangular pulse function,

$$p_{W_n}(z) = \begin{cases} \frac{1}{q_n}, & \frac{-q_n}{2} < z < \frac{q_n}{2} \\ 0, & \text{elsewhere} \end{cases} \tag{21}$$

whose width depends on the quantizer step-size (q_n) defined in Section II, and $c_{\delta'_n}(z)$ denotes the impulse train

$$c_{\delta'_n}(z) = \sum_{i_n \in \mathbb{Z}} q_n \delta'\left(z - i_n q_n - \frac{q_n}{2}\right). \tag{22}$$

The "$\star$" in (20) denotes the convolution operation, and $\delta'(\cdot)$ in (22) is the Dirac-delta function. This process of convolution followed by conventional sampling is popularly known as "area sampling" [27]. Also, note that $p_{W_n}(\cdot)$ is also the pdf of a uniform random variable$W_n \sim \mathcal{U}\left(-\frac{q_n}{2}, \frac{q_n}{2}\right)$. Thus, quantization introduces two "types" of distortions or errors: a) the additive uniform noise (AUN) error, and b) the aliasing error due to sampling.

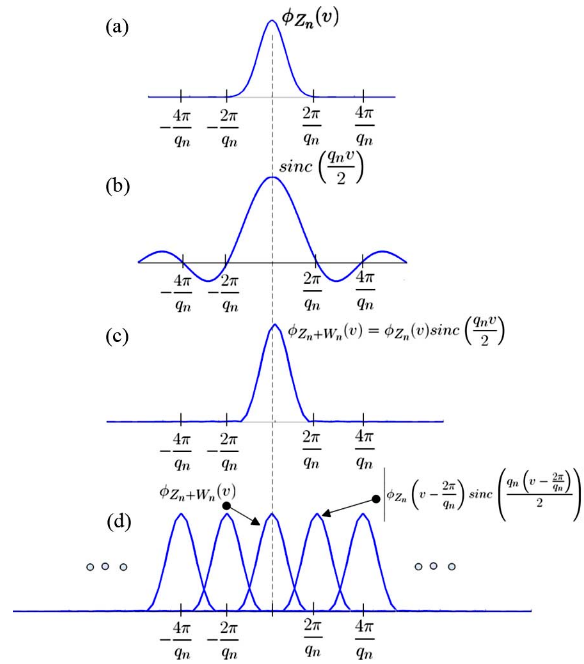


Fig. 4. Illustration of the quantization process in the CF domain (a) CF of Z_n (b) CF of W_n, the sinc function (c) CF of $Z_n + W_n$ (d) Repetition of CF of $Z_n + W_n$; the CF of the quantized variable is given by the summation of these repetitions after weighting each appropriately (see (24)).

The two errors introduced due to quantization can be better visualized in the characteristic function (CF) domain. The CF of a random variable X is obtained by taking the Fourier transform of its pdf $p_X(x)$

$$\phi_X(v) = \int_{-\infty}^{\infty} p_X(x) e^{jvx} dx = E\left[e^{jvx}\right]. \tag{23}$$

Taking the Fourier transform of (20), one obtains the CF of output variable U_n

$$\begin{aligned} \phi_{U_n}(v) = & \sum_{i_n=-\infty}^{\infty} \phi_{Z_n}\left(v + i_n \frac{2\pi}{q_n}\right) \\ & \times \operatorname{sinc}\left(\frac{q_n(v + i_n \frac{2\pi}{q})}{2}\right) e^{-j i_n \frac{2\pi}{q_n}\frac{q_n}{2}} \\ = & \sum_{i_n=-\infty}^{\infty} (-1)^{i_n} \phi_{Z_n}\left(v + i_n \frac{2\pi}{q_n}\right) \\ & \times \operatorname{sinc}\left(\frac{q_n(v + i_n \frac{2\pi}{q})}{2}\right) \end{aligned} \tag{24}$$

where $\phi_{Z_n}(v)$ is the CF of the input Z_n and $\operatorname{sinc}(v) = \frac{sin(v)}{v}$. Note that (24) is different from the one in [17, p. 65, eq. (4.11)] as we have considered a mid-rise quantizer here instead of a mid-tread quantizer used in [17]. Fig. 4 shows the operations in the "frequency" domain. Note that the central lobe ($i_n = 0$ in (24))

$$\phi_{Z_n+W_n}(v) = \phi_{Z_n}(v) \cdot \operatorname{sinc}\left(\frac{q_n v}{2}\right) \tag{25}$$

corresponds to the CF one would obtain by adding an independent and uniformly distributed random variable W_n to the input Z_n. It is clear from Fig. 4 that, in addition to the error introduced due to the addition of uniform noise, quantization also causes an aliasing error due to overlapping (and phase shifted) lobes of $\phi_{Z_n+W_n}(v)$. However, if the input pdf is band-limited so that $\phi_{Z_n}(v) = 0$ for $|v| > \frac{\pi}{q_n}$, then the "frequency"-shifted versions of $\phi_{Z_n+W_n}(v)$ do not overlap and, in principle, the original pdf can be reconstructed from the knowledge of $p_{U_n}(\cdot)$. This is Widrow's first quantization theorem:

Theorem 3. (Widrow's Quantization Theorem I): If the CF of the input variable Z_n is bandlimited so that

$$\phi_{Z_n}(v) = 0, \quad |v| > \frac{\pi}{q_n} \tag{26}$$

then the different lobes in $\phi_{U_n}(v)$ do not overlap, and in principle, the original pdf $p_n(z_n)$ (before quantization) can be recovered from the pdf of U_n.

When $\phi_{Z_n}(v) = 0$ for $|v| > \frac{2\pi}{q_n}$ so that the derivatives of $\phi_{U_n}(v)$ computed at $v = 0$ are not affected due to the overlap of CF lobes, then the moments of Z_n can be recovered from those of U_n. This is Widrow's second theorem.

Theorem 4. (Widrow's Quantization Theorem II): If the CF of Z_n is bandlimited so that

$$\phi_{Z_n}(v) = 0, \quad |v| > \frac{2\pi}{q_n} \tag{27}$$

then the moments of Z_n can be derived from the moments of U_n.

In the following, we assume that Theorem 3 (and hence Theorem 4) holds, and derive a rule to fuse multilevel decisions at the fusion center. We also note here that Widrow's additive model for quantization noise, and, hence the fusion rule derived in the next section, is better suited for high resolution quantization (See [28] and references therein).

B. Derivation of a Computationally Efficient Fusion Rule

As discussed previously, the high complexity in computing the mGLRT statistic for quantized observations stems from the need for computing multidimensional integrals. We propose to simplify the fusion process by adding *controlled noise* to the observations received at the fusion center. The system is shown in Fig. 3. An externally generated noise, d_n, with pdf $p_{D_n}(d_n)$ is added to the quantized observations from each sensor n before fusing them for making a global decision. Denote the new observations by $y_n = u_n + d_n$ whose CF is given by

$$\phi_{Y_n}(v) = \phi_{U_n}(v) \cdot \phi_{D_n}(v). \tag{28}$$

One can choose the noise source with a bandlimited CF to filter out the repeated and phase-shifted CF lobes in $\phi_{U_n}(v)$. This is analogous to low-pass filtering in signal processing. We, therefore, call the noise D_n, the LPF-noise. As shown in Fig. 4(d), an ideal noise source would be one with a rectangular CF in the passband, $-\frac{\pi}{q_n} \le v \le \frac{\pi}{q_n}$, (also see Fig. 5). However, a rectangular function in the CF domain corresponds to a pdf whose shape corresponds to a sinc function, an invalid pdf. Note that this is similar to the nonrealizability of an ideal low-pass filter in signal processing. One, therefore, needs to carefully design D_n so that it causes minimal distortion while transforming the discrete-valued random variable, U_n, into a continuous variable, Y_n. As long as the input variable Z_n satisfies Widrow's first quantization theorem (Theorem 3) under both H_1 and H_0, we have,

$$Y_n = Z_n + W_n + D_n. \tag{29}$$

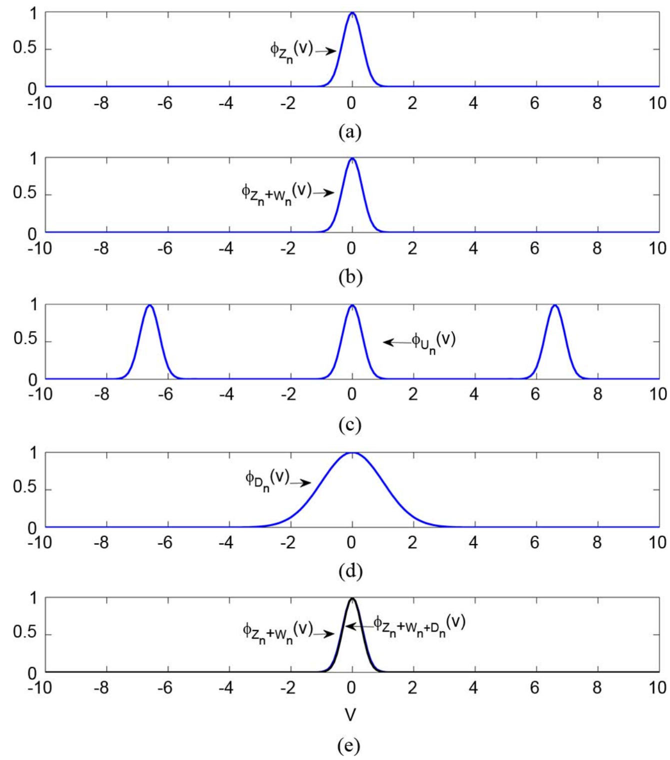


Fig. 5. "Filtering" the quantized signal with LPF-noise. The quantization step size, q_n, is set to $0.3\sigma_n$. (a) CF of Z_n; (b) CF of $Z_n + W_n$; (c) CF of U_n; (d) CF of the external LPF-noise, D_n; (e) CF of $Y_n = Z_n + W_n + D_n$.

Thus, under hypothesis H_1, the pdf of data, y_{nl}, at time instant l is

$$\begin{aligned} p_{Y_n}(y_{nl}; \psi_n, H_1) &= p_{Z_n}(y_{nl}; H_1) \star p_{W_n}(y_{nl}) \star p_{D_n}(y_{nl}) \\ &= f_n(y_{nl}; \psi_n) \star p_{W_n}(y_{nl}) \star p_{D_n}(y_{nl}). \end{aligned} \tag{31}$$

Using a copula density (say) $k_1(\cdot; \psi_d) \in \mathcal{A}_k$ to estimate the dependence structure between sensor observations, the joint pdf of the data $\mathbf{y}_l = (y_{1l}, y_{2l}, \ldots, y_{Nl})$ can now be approximated as

$$\begin{aligned} \hat{p}_{\mathbf{Y}}(y_{1l}, \ldots, y_{Nl}; \psi, H_1) = & \left\{ \prod_{n=1}^{N} p_{Y_n}(y_{nl}; \psi_n, H_1) \right\} \\ & \times k_1 \left\{ F_{Y_1}(y_{1l}; \psi_1), \ldots, F_{Y_N}(y_{Nl}; \psi_N); \psi_d \right\} \end{aligned} \tag{32}$$

where

$$F_{Y_n}(y) = \int_{-\infty}^{y} p_{Y_n}(t; \psi_n, H_1) dt \tag{33}$$

denotes the cdf of Y_n under H_1.

Similarly, the joint pdf of the data under H_0 can be approximated as

$$\begin{aligned} \hat{p}_{\mathbf{Y}}(y_{1l}, \ldots, y_{Nl}; \lambda, H_0) = & \left\{ \prod_{n=1}^{N} p_{Y_n}(y_{nl}; \lambda_n, H_0) \right\} \\ & \times k_0 \left\{ G_{Y_1}(y_{1l}; \lambda_1), \ldots, G_{Y_N}(y_{Nl}; \lambda_N); \lambda_d \right\} \end{aligned} \tag{34}$$

where

$$\begin{aligned} p_{Y_n}(y_{nl}; \lambda_n, H_0) &= p_{Z_n}(y_{nl}; H_0) \star p_{W_n}(y_{nl}) \star p_{D_n}(y_{nl}) \\ &= g_n(y_{nl}; \lambda_n) \star p_{W_n}(y_{nl}) \star p_{D_n}(y_{nl}) \end{aligned} \tag{35}$$

$k_0(\cdot;\lambda_d) \in \mathcal{A}_k$ is the copula density used to estimate the dependence structure of sensor observations under H_0, and

$$G_{Y_n}(y) = \int_{-\infty}^{y} p_{Y_n}(t;\lambda_n, H_0)dt \tag{36}$$

denotes the cdf of Y_n when the underlying hypothesis is H_0. With (32) and (34), it is now straightforward to derive the (misspecified) GLRT

$$\mathcal{T}_k'(\mathbf{y}) = \log \frac{\max\limits_{k_1(\cdot)\in\mathcal{A}_k,\Psi} \prod_{l=1}^{L} p_{\mathbf{Y}}(y_{1l},\ldots,y_{Nl};\psi,H_1)}{\max\limits_{k_0(\cdot)\in\mathcal{A}_k,\Lambda} \prod_{l=1}^{L} p_{\mathbf{Y}}(y_{1l},\ldots,y_{Nl};\lambda,H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \tag{37}$$

The test derived above involves continuous-valued variables and thus does not involve computation of multidimensional integrals. This greatly simplifies the test. The reduced complexity is, however, at the expense of decreased SNR due to the injection of noise d_n at the fusion center. The addition of external noise facilitates *filtering* of the baseband CF, $\phi_{Z_n+W_n}(v)$, from the received quantized observations $\phi_{U_n}(v)$. This noise should be designed so that it destroys as little information as possible while filtering the required signal.

Next, we present a numerical illustration.

VI. An Illustrative Example

In this section, we consider the problem of detecting a random phenomenon using a network of two sensors. It is known that the observations received at the local quantizers each follow a Gaussian distribution. That is

$$\begin{aligned} H_0: &\quad Z_1 \sim \mathcal{N}(0,\sqrt{10}), \quad Z_2 \sim \mathcal{N}(0,\sqrt{10}) \\ H_1: &\quad Z_1 \sim \mathcal{N}(\mu_1,\sqrt{10}), \quad Z_2 \sim \mathcal{N}(\mu_2,\sqrt{10}) \end{aligned} \tag{38}$$

where

$$\mathcal{N}(\mu,\sigma) := \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-1}{2\sigma^2}(z-\mu)^2\right)$$

is the usual univariate Gaussian density function. The means, μ_1 and μ_2, under the hypothesis, H_1, are unknown, although *a priori* we know that they are greater than zero. Further, the observations may be statistically dependent; however, no knowledge about the dependence structure (and hence, the joint distribution) is provided.

The observations, $\{z_{1l}, z_{2l}\}_{l=1}^{L}$, at the two local sensors are passed through uniform scalar quantizers before their transmission to the fusion center. Thus, the fusion center has access only to the quantized measurements, $\mathbf{u} = \{u_{1l}, u_{2l}\}_{l=1}^{L}$, to make a global decision in favor of one of the two hypotheses. Then, the GLRT based fusion rule for this problem is the same as the one derived in (14), with the joint probabilities

$$\begin{aligned} &\hat{P}^*_{i_1,i_2}\left(\hat{\mu}_1,\hat{\mu}_2,\hat{\psi}_d\right) \\ &= K_1^*\left\{\Phi_{\hat{\mu}_1,\sqrt{10}}\left((i_1+1)q_1\right),\right. \\ &\qquad \left.\Phi_{\hat{\mu}_2,\sqrt{10}}\left((i_2+1)q_2\right);\hat{\psi}_d\right\} \\ &\quad - K_1^*\left\{\Phi_{\hat{\mu}_1,\sqrt{10}}\left(i_1q_1\right),\Phi_{\hat{\mu}_2,\sqrt{10}}\left(i_2q_2\right);\hat{\psi}_d\right\} \end{aligned} \tag{39}$$

and

$$\begin{aligned} \hat{Q}^*_{i_1,i_2} &= \Phi_{0,\sqrt{10}}\left((i_1+1)q_1\right)\cdot\Phi_{0,\sqrt{10}}\left((i_2+1)q_2\right) - \\ &\quad \Phi_{0,\sqrt{10}}\left(i_1q_1\right)\Phi_{0,\sqrt{10}}\left(i_2q_2\right) \end{aligned} \tag{40}$$

where $\Phi_{\mu,\sigma}(\cdot)$ denotes the Gaussian cdf with mean, μ, and standard deviation, σ. The unknown marginal parameters, μ_1 and μ_2, and the copula function $K_1^*(\cdot;\hat{\psi}_d)$ in (39) are obtained by maximizing the generalized likelihood ratio as shown in (14). An alternative computationally efficient test was derived in Section V-C which involves injection of LPF-noise before fusion. We evaluate its performance using the example presented here.

Although Gaussian CFs are not perfectly bandlimited, a property necessary for using the LPF-noise based fusion rule, they are very close to being bandlimited for all practical purposes. Fig. 5 shows quantization and the effect of LPF-noise in the CF domain. The quantization step size, q_n is set to 0.3 of the input standard deviation ($q_n = 0.3\sigma_n$). The CF of the input variable z_n is shown in Fig. 5(a). Addition of the quantization noise, w_1, is equivalent to multiplication of $\phi_{Z_n}(v)$ (shown in Fig. 5(a)) with a sinc function, $\mathrm{sinc}\left(\frac{q_n v}{2}\right)$. The resultant CF, $\phi_{Z_n+W_n}(v)$, is shown in Fig. 5(b). This CF is repeated and summed in Fig. 5(c) which represents the CF of the quantized signal, u_n (see (24)). The CF of the LPF-noise, D_n, a standard Gaussian distributed variable in this example, is shown in Fig. 5(d).[1] It is clear that multiplication of $\phi_{U_n}(v)$ with $\phi_{D_n}(v)$ which is equivalent to addition of d_n to $z_n + w_n$ in the random variable domain, "filters" the signal so that only the main lobe ($v = 0$) of $\phi_{U_n}(v)$ is retained (Fig. 5(e)). Since the LPF-noise is different from the ideal one with rectangular CF, the signal, $z_n + w_n$, undergoes some distortion while being "filtered." However, this distortion is almost imperceptible as evident from Fig. 5(e).

The pdf of the transformed variable, $Y_n = Z_n + W_n + D_n$, under the hypothesis H_1 is given by

$$\begin{aligned} &p_{Y_n}(y_n;\mu_n,H_1) \\ &\quad = p_{Z_n+D_n}(y_n) \star p_{W_n}(y_n) \\ &\quad = \mathcal{N}\left(\mu_n,\sqrt{\sigma_n^2+\sigma_{d_n}^2}\right) \star \mathcal{U}\left(-\frac{q_n}{2},\frac{q_n}{2}\right) \\ &\quad = \frac{1}{q_n}\left[\Phi_{\mu_n,\sqrt{11}}\left(y_n+\frac{q_n}{2}\right) - \Phi_{\mu_n,\sqrt{11}}\left(y_n-\frac{q_n}{2}\right)\right]. \end{aligned} \tag{41}$$

Similarly, under H_0, we have

$$\begin{aligned} &p_{Y_n}(y_n,H_0) \\ &\quad = p_{Z_n+D_n}(y_n) \star p_{W_n}(y_n) \\ &\quad = \mathcal{N}\left(0,\sqrt{\sigma_n^2+\sigma_{d_n}^2}\right) \star \mathcal{U}\left(-\frac{q_n}{2},\frac{q_n}{2}\right) \\ &\quad = \frac{1}{q_n}\left[\Phi_{0,\sqrt{11}}\left(y_n+\frac{q_n}{2}\right) - \Phi_{0,\sqrt{11}}\left(y_n-\frac{q_n}{2}\right)\right]. \end{aligned} \tag{42}$$

Having derived the marginal pdfs ((41) and (42)), the LPF-noise-based fusion rule (37) is now applied for testing between the two hypotheses. We include the Frank and the Gaussian copula functions in the set, $\mathcal{A}_k$, of potential copula models for characterizing dependence between observations under H_1.

In order to inject dependence between observations under H_1, we first generate dependent uniformly distributed bivariate

[1] It is important to note that a standard Gaussian noise may not be the "best" LPF-noise. It is used here to provide a simple illustrative example.

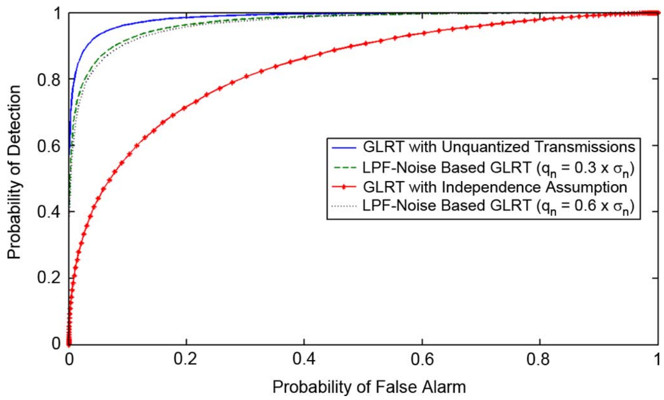


Fig. 6. Monte Carlo-based receiver operating characteristics. Performance of the fusion rule based on LPF noise is very close to the upper bound given by the analog transmission case. Also, the LPF-noise-based GLRT outperforms the one designed assuming statistical independence between the observations.

samples, $\mathbf{V} = \{(v_{1l}, v_{2l})\}_{l=1,2,\ldots,L}$, using Clayton copula with Kendall's τ set to 0.31. The inverse cdf corresponding to each sensor's observation (specified in (38)) is then used to transform the bivariate samples, $\mathbf{V}$, to give a bivariate vector of dependent sensor observations with the required marginals:

$$z_{nl} = \Phi^{-1}_{\mu_n,\sigma_n}(v_{nl}), \quad \forall n, l. \tag{43}$$

The marginal parameters, μ_1 and μ_2, are set to 0.5. Detection performance of the LPF-noise based GLRT is evaluated using this synthetic dataset. As discussed earlier, the set, $\mathcal{A}_k$, of potential copula functions consists of the Frank and Gaussian copula functions. Note that we have deliberately excluded the Clayton copula from this set so that we can evaluate the detection performance when the true underlying copula is unavailable. Hence, we also call the test the misspecified GLRT.

In Fig. 6, we plot the ROC curves using 50 000 Monte Carlo trials. The decision window, L, is set to 50 samples. That is, we assume that the sensors observe the phenomenon over $L = 50$ time intervals before making a decision in favor of either hypothesis. It is evident from the figure that the performance of the LPF-noise based fusion rule is very close to the upper bound given by the analog/unquantized transmission case albeit with reduced computational complexity. This is true for both the quantizers, $q_n = 0.3\,\sigma_n$ and $q_n = 0.6\,\sigma_n$, considered here. The quantization step sizes of $0.3\,\sigma_n$ and $0.6\,\sigma_n$ correspond to 22 and 12 quantization levels, respectively, in the $[-3\sigma, 3\sigma]$ region of a Gaussian density function.

Another approach that is often adopted to address the issue of computational complexity is to deliberately neglect statistical dependence between sensor observations while designing the test. The test, so designed, would require computation of N one-dimensional integration operations as opposed to N-dimensional integrations where N is the number of sensors. However, such an approach severely degrades the detection performance as evident from Fig. 6. The LPF noise based GLRT significantly outperforms the one designed with the statistical independence assumption.

VII. Determination of the Detector Threshold

Following the Neyman-Pearson formulation, we now look for a method to set the detector threshold η in (14) so that the false alarm probability, P_F, is constrained to $\alpha \in (0,1)$. This, however, requires the knowledge of $p_{\mathcal{T}_k}(t_k; H_0)$, the pdf of the test statistic under the null hypothesis. Since the postulated statistical models, $\{\hat{f}(\mathbf{z}; \Psi \subset \mathbb{R}^a)\}$ and $\{\hat{g}(\mathbf{z}; \Lambda \subset \mathbb{R}^b)\}$, under H_1 and H_0, respectively, are only approximations of the true underlying distributions, it is difficult to derive the exact distribution of the test statistic under either hypothesis. However, some advancement is possible for a certain class of problems especially when L is large. The following theorem, due to Wilks [29], identifies this class of problems.

Theorem 5: Suppose the following conditions hold, in addition to the usual regularity conditions [30] that ensure the validity of asymptotic ML theory:

C 1. $\{\hat{f}(\mathbf{z}; \Psi \subset \mathbb{R}^a)\}$ and $\{\hat{g}(\mathbf{z}; \Lambda \subset \mathbb{R}^b)\}$ are well-specified under H_0,

C 2. $\{\hat{g}(\mathbf{z}; \Lambda)\}$ is nested in $\{\hat{f}(\mathbf{z}; \Psi)\}$, i.e., $\hat{g}(\mathbf{z}; \lambda) \in \{\hat{f}(\mathbf{z}; \Psi)\}$, $\forall \lambda \in \Lambda$.

Then the modified test statistic, $2\mathcal{T}_k(\cdot)$, converges in distribution ($\xrightarrow{d}$) to a chi-squared distribution with $\nu\ (= a - b)$ degrees of freedom under the null hypothesis.

$$2\mathcal{T}_k(\mathbf{z}) \xrightarrow{d} \chi^2_{a-b}, \quad \text{under } H_0. \tag{44}$$

From (44), the probability of false alarm, P_F, is

$$\begin{aligned} P_F &= Pr\left(2\mathcal{T}_k(\cdot) > 2\eta; H_0\right) \\ &\overset{L\to\infty}{=} Q_{\chi^2_{a-b}}(2\eta) \end{aligned} \tag{45}$$

where $Q_{\chi^2_\nu}(\cdot)$ denotes the right-tail probability of a chi-squared random variable with ν degrees of freedom. One can thus obtain the threshold η so that P_F is constrained to a desired level $\alpha \in (0,1)$ as follows:

$$\eta = \frac{Q^{-1}_{\chi^2_{a-b}}(\alpha)}{2}. \tag{46}$$

The assumption of a well-specified $g(\cdot; \lambda)$ is reasonable for many applications. For example, it is always possible to collect enough training data under H_0 (when there is no signal present), so that $g(\cdot)$ can be consistently estimated.

Note that the illustrative example in Section VI satisfies the conditions of Theorem 5, and thus belongs to the class of problems for which the detector threshold can be determined. Here, the true distribution under H_0, given by the product of two univariate Gaussian pdfs, was assumed to be completely known (and hence well-specified). Now, given that the data belongs to the null hypothesis

$$\begin{aligned} \hat{f}(\mathbf{z}; \hat{\psi}) &= \left(\prod_{n=1}^{2} f(z_n; \hat{\psi}_n)\right) k(\cdot; \hat{\psi}_d) \xrightarrow{H_0} g(\mathbf{z}; \lambda) \\ &= \prod_{n=1}^{2} g(z_n; \lambda_n) \end{aligned} \tag{47}$$

since both the Frank and the Gaussian copulas converge to the independence copula. Thus, condition $\mathbf{C1}$ of Theorem 5 holds. Also, it is easy to see that $g(\mathbf{z}; \Lambda)$ is nested in the family defined by $\{\hat{f}(\mathbf{z}; \Psi)\}$ since the marginal pdfs under both hypotheses are univariate Gaussian pdfs. Thus, the condition $\mathbf{C2}$ is satisfied, and we have the asymptotic convergence of the test statistic to a chi-squared distribution with $a - b = 3$ degrees of freedom, where the number of unknown parameters under H_1, denoted

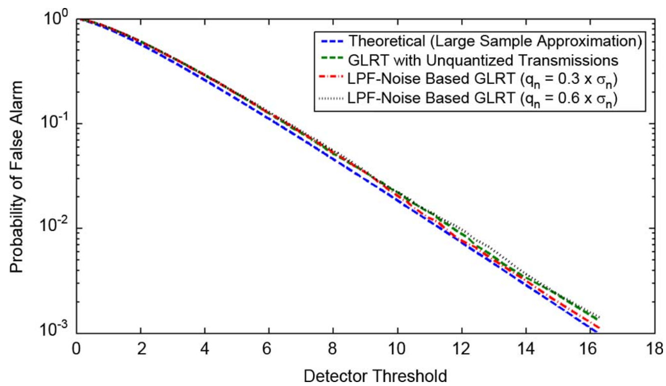


Fig. 7. System probability of false alarm versus Detector threshold. A good match between the theoretical and simulated P_F values is evident from the figure.

by "a" is three (μ_1, μ_2 and ψ_d), and that under H_0, denoted by "b," is zero. Thus, $P_F = Q_{\chi_3^2}(2\eta)$. Fig. 7 shows a plot of this theoretical P_F along with simulated false alarm values obtained using 50 000 Monte Carlo trials with the decision window, L, set to 50 samples.

A good match between the theoretical and simulated P_F values across the two different quantization step sizes is evident from the figure. Note that the distribution of $2\mathcal{T}_k(\mathbf{z}; \hat{\lambda}, \hat{\psi})$ under the null hypothesis depends only on the model complexities of $\{\hat{f}(\cdot; \Psi)\}$ and $\{g(\cdot; \Lambda)\}$, i.e., the number of uncertain parameters a and b. If the set $\mathcal{A}_k$ consists of copula densities with parameters of different dimensions, the threshold η must be adjusted accordingly (see (46)) to maintain a desired false alarm probability. Alternatively, one could restrict the set $\mathcal{A}_k$ to include copula models with equal complexity to avoid the extra step of varying η in real time.

VIII. Conclusion

In this paper, the problem of fusing statistically dependent sensor decisions for the detection of a random event was considered. Sensor observations (or features extracted thereof) are first quantized using uniform multilevel quantizers before their transmission to the fusion center. Intermodal dependence was assumed to be unknown and was approximated using copula functions. A GLRT-based decision fusion algorithm that can fuse both hard and soft local decisions was derived. The important problem of selecting the best copula was embedded in the GLRT formulation. It was noted that the derived copula-based fusion algorithm becomes computationally expensive as the number of sensors and/or number of quantization levels increase. A novel approach based on Widrow's additive quantization noise model was developed which requires deliberate injection of an external noise at the receiver before fusion. The addition of external noise at the fusion center effectively "filters" the baseband CFs by rejecting the repetitive CF lobes that arise due to quantization. Since this process is analogous to low-pass filtering (LPF) in signal processing, we term this noise, the LPF-noise.

An illustrative example, using different copula functions such as the Clayton, Frank, and Gaussian copulas was presented. Gaussian noise sources were used to generate LPF-noise at the fusion center, and results for two different quantization step sizes were obtained. Our results show that the approach based on LPF-noise can be considerably accurate provided the CF of the input signals are bandlimited and Widrow's first quantization theorem is satisfied. The key to the success of this computationally efficient approach is the choice of the external noise source used for filtering the baseband CF. Design of a noise source that introduces minimal distortion while filtering is a topic of future research.

References

[1] P. K. Varshney, *Distributed Detection and Data Fusion*. New York: Springer-Verlag, 1997.

[2] J.-F. Chamberland and V. V. Veeravalli, "Wireless sensors in distributed detection applications," *IEEE Signal Process. Mag.*, vol. 24, no. 3, pp. 16–25, 2007.

[3] I. Y. Hoballah and P. K. Varshney, "An information theoretic approach to the distributed detection problem," *IEEE Trans. Inf. Theory*, vol. 35, no. 5, pp. 988–994, 1989.

[4] B. Chen, L. Tong, and P. K. Varshney, "Channel-aware distributed detection in wireless sensor networks," *IEEE Signal Process. Mag.*, vol. 23, no. 4, pp. 16–26, 2006.

[5] I. Y. Hoballah and P. K. Varshney, "Distributed Bayesian signal detection," *IEEE Trans. Inf. Theory*, vol. 35, no. 5, pp. 995–1000, 1989.

[6] S. C. A. Thomopoulos, R. Viswanathan, and D. K. Bougoulias, "Optimal distributed decision fusion," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 25, no. 5, pp. 761–765, 1989.

[7] V. Aalo and R. Viswanathan, "On distributed detection with correlated sensors: Two examples," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 25, no. 3, pp. 414–421, 1989.

[8] P. Willett, P. F. Swaszek, and R. S. Blum, "The good, bad and ugly: Distributed detection of a known signal in dependent Gaussian noise," *IEEE Trans. Signal Process.*, vol. 48, no. 12, pp. 3266–3279, 2000.

[9] J. Tsitsiklis and M. Athans, "On the complexity of decentralized decision making and detection problems," *IEEE Trans. Autom. Control*, vol. 30, no. 5, pp. 440–446, 1985.

[10] C. H. Papadimitriou and J. N. Tsitsiklis, "Intractable problems in control theory," in *Proc. 24th IEEE Conf. Dec. Control*, 1985, vol. 24, pp. 1099–1103.

[11] E. Drakopoulos and C.-C. Lee, "Optimum multisensor fusion of correlated local decisions," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 27, no. 4, pp. 593–606, 1991.

[12] M. Kam, Q. Zhu, and W. S. Gray, "Optimal data fusion of correlated local decisions in multiple sensor detection systems," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 28, no. 3, pp. 916–920, 1992.

[13] S. G. Iyengar, P. K. Varshney, and T. Damarla, "A parametric copula-based framework for hypothesis testing using heterogeneous data," *IEEE Trans. Signal Process.*, vol. 59, no. 5, pp. 2308–2319, 2011.

[14] A. Sundaresan, P. K. Varshney, and N. S. V. Rao, "Copula-based fusion of correlated decisions," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 47, no. 1, pp. 454–471, 2011.

[15] R. B. Nelsen, *An Introduction to Copulas*. New York: Springer-Verlag, 1999.

[16] F. Gustafsson and R. Karlsson, "Statistical results for system identification based on quantized observations," *Automatica*, vol. 45, no. 12, pp. 2794–2801, 2009.

[17] B. Widrow and I. Kollar, *Quantization Noise: Roundoff Error in Digital Computation, Signal processing, Control, and Communications*. Cambridge, U.K.: Cambridge Univ. Press, 2008.

[18] H. White, *Estimation, Inference and Specification Analysis*. Cambridge, U.K.: Cambridge Univ. Press, 1994.

[19] H. Joe, *Multivariate Dependence and Related Concepts*. Durham, NC: Chapman & Hall, 1997.

[20] D. Kurowicka and R. Cooke, *Uncertainty Analysis With High Dimensional Dependence Modeling*. New York: Wiley, 2006.

[21] H. Joe and J. J. Xu, "The estimation method of inference functions for margins for multivariate models," Univ. of British Columbia, Vancouver, B.C., Canada, 1996.

[22] L. Roberts, "Picture coding using pseudo-random noise," *IRE Trans. Inf. Theory*, vol. 8, no. 2, pp. 145–154, 1962.

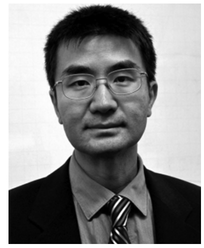
[23] L. Schuchman, "Dither signals and their effect on quantization noise," *IEEE Trans. Commun. Technol.*, vol. 12, no. 4, pp. 162–165, 1964.

[24] R. M. Gray and J. T. G. Stockham, “Dithered quantizers,” *IEEE Trans. Inf. Theory*, vol. 39, no. 3, pp. 805–812, 1993.
[25] H. Chen, P. K. Varshney, S. M. Kay, and J. H. Michels, “Theory of the stochastic resonance effect in signal detection: Part I—Fixed detectors,” *IEEE Trans. Signal Process.*, vol. 55, no. 7, pp. 3172–3184, 2007.
[26] B. Widrow, “A study of rough amplitude quantization by means of Nyquist sampling theory,” *IRE Trans. Circuit Theory*, vol. 3, no. 4, pp. 266–276, 1956.
[27] B. Widrow, I. Kollar, and M.-C. Liu, “Statistical theory of quantization,” *IEEE Trans. Instrum. Meas.*, vol. 45, no. 2, pp. 353–361, 1996.
[28] R. M. Gray and D. L. Neuhoff, “Quantization,” *IEEE Trans. Inf. Theory*, vol. 44, no. 6, pp. 2325–2383, 1998.
[29] S. S. Wilks, “The large sample distribution of the likelihood ratio for testing composite hypotheses,” *Ann. Math. Statist.*, vol. 9, pp. 60–62, 1938.
[30] A. V. van der, *Asymptotic Statistics*. Cambridge, U.K.: Cambridge Univ. Press, 1998.

Satish G. Iyengar received the B. E. degree in Electronics and Telecommunication from the University of Mumbai, Mumbai, India, in 2002. He received the M.S. and Ph.D. degrees in electrical engineering from Syracuse University, Syracuse, NY, in 2004 and 2011, respectively.

He is currently a Research Engineer at the GE Global Research Corporation, Niskayuna, NY. Prior to joining GE, he was a Postdoctoral Research Associate in the Sensor Fusion Laboratory at Syracuse University. During 2003–2011, he has held teaching and research assistantships in the departments of electrical engineering and computer science, and bioengineering and neuroscience. He was a research intern at the Blue Highway LLC (then a WelchAllyn company), in Syracuse, NY in 2010–2011, a visiting researcher at the U.S. Army Research Laboratory, Adelphi, MD, under the Scientific Outreach for Army Research (SOAR) program in 2009, and a student intern in the department of astronomy and astrophysics at the Tata Institute of Fundamental Research, Mumbai, India, in 2001–2002.

Dr. Iyengar is a recipient of the Syracuse University “All-University Doctoral Prize,” 2011. His current research interests are in statistical signal processing, distributed sensor networks and information fusion, and machine diagnostics and prognostics.



Ruixin Niu (M’04–SM’11) received the B.S. degree from Xi’an Jiaotong University, Xi’an, China, in 1994, the M.S. degree from the Institute of Electronics, Chinese Academy of Sciences, Beijing, China, in 1997, and the Ph.D. degree from the University of Connecticut, Storrs, in 2001, all in electrical engineering.

He is currently an assistant professor with the Department of Electrical and Computer Engineering, at Virginia Commonwealth University (VCU), Richmond. Before joining VCU, he was a research assistant professor with Syracuse University, Syracuse, NY. His research interests are in the areas of statistical signal processing and its applications, including detection, estimation, data fusion, sensor networks, communications, and image processing.

Dr. Niu received the Best Paper Award, at the Seventh International Conference on Information Fusion in 2004. He is a coauthor of the paper that won the Best Student Paper Award at the Thirteenth International Conference on Information Fusion in 2010. He is an Associate Editor of IEEE TRANSACTIONS ON SIGNAL PROCESSING, the Associate Administrative Editor of the *Journal of Advances in Information Fusion*, and an Associate Editor of the *International Journal of Distributed Sensor Networks*.

Pramod K. Varshney (S’72–M’77–SM’82–F’97) was born in Allahabad, India, on July 1, 1952. He received the B.S. degree in electrical engineering and computer science (with highest honors), and the M.S. and Ph.D. degrees in electrical engineering from the University of Illinois at Urbana-Champaign in 1972, 1974, and 1976, respectively.

From 1972 to 1976, he held teaching and research assistantships at the University of Illinois. Since 1976, he has been with Syracuse University, Syracuse, NY, where he is currently a Distinguished Professor of Electrical Engineering and Computer Science and the Director of CASE: Center for Advanced Systems and Engineering. He served as the Associate Chair of the department from 1993 to 1996. He is also an Adjunct Professor of Radiology at Upstate Medical University, Syracuse, NY. His current research interests are in distributed sensor networks and data fusion, detection and estimation theory, wireless communications, image processing, radar signal processing, and remote sensing. He has published extensively. He is the author of *Distributed Detection and Data Fusion* (Springer-Verlag, 1997). He has served as a consultant to several major companies.

Dr. Varshney was a James Scholar, a Bronze Tablet Senior, and a Fellow while at the University of Illinois. He is a member of Tau Beta Pi and is the recipient of the 1981 ASEE Dow Outstanding Young Faculty Award. He was elected to the grade of Fellow of the IEEE in 1997 for his contributions in the area of distributed detection and data fusion. He was the Guest Editor of the Special Issue on Data Fusion of the PROCEEDINGS OF THE IEEE, January 1997. In 2000, he received the Third Millennium Medal from the IEEE and Chancellor’s Citation for exceptional academic achievement at Syracuse University. He is the recipient of the IEEE 2012 Judith A. Resnik Award. He serves as a Distinguished Lecturer for the IEEE Aerospace and Electronic Systems (AES) Society. He is on the Editorial Board of the *Journal on Advances in Information Fusion*. He was the President of International Society of Information Fusion during 2001.