☞ Quiz: Sections 6.1 & 6.2 | Linear Algebra MATH 310 R. Hammack | December 1, 2016

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Directions: Please answer in the space provided. No calculators. Please put all phones, etc., away.

1. Suppose T is a linear transformation with matrix $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$.

   (a) State the domain of T. $\boxed{\mathbb{R}^3}$

   (b) State the codomain of T. $\boxed{\mathbb{R}^2}$

   (c) Find a basis for the kernel of T.

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} x - y + 2z = 0 \\ \quad\ \ y + 2z = 0 \end{cases}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix} \xrightarrow{R_1 + R_2 \to R_1} \begin{bmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad \begin{matrix} x = -4z \\ y = -2z \end{matrix}$$

$$\ker(T) = \left\{ \begin{bmatrix} -4z \\ -2z \\ z \end{bmatrix} : z \in \mathbb{R} \right\} = \left\{ z\begin{bmatrix} -4 \\ -2 \\ 1 \end{bmatrix} : z \in \mathbb{R} \right\} = \text{span}\left(\begin{bmatrix} -4 \\ -2 \\ 1 \end{bmatrix}\right)$$

$$\boxed{\text{Basis for } \ker(T) \text{ is } \quad \mathcal{B} = \left\{ \begin{bmatrix} -4 \\ -2 \\ 1 \end{bmatrix} \right\}}$$

   (d) nullity(T) = $\dim(\text{Ker}(T)) = \boxed{1}$

   (e) rank(T) = $3 - \text{nullity}(T) = \boxed{2}$

   (f) Is T one-to-one? $\boxed{\text{NO}}$ because $\ker(T) \neq \{\vec{0}\}$

   (g) Is T onto? $\boxed{\text{YES}}$ because $\text{rank}(T) = 2 =$ (dimension of codomain.)

   (h) State the range of T. $\boxed{\mathbb{R}^2}$

2. Suppose $S : \mathbb{R}^4 \to \mathbb{R}^6$ is a linear transformation, and $\text{rank}(S) = 3$. What is the nullity of S? Explain.

$$\text{rank}(S) + \text{nullity}(S) = \dim(\mathbb{R}^4)$$

$$3 + \text{nullity}(S) = 4 \quad \Rightarrow \boxed{\text{nullity}(S) = 1}$$