

Name: Richard

MATH 310

Score: 10

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Directions: Please answer in the space provided. No calculators. Please put all phones, etc., away.

The following questions concern the matrix $A = \begin{bmatrix} 1 & 3 & -2 & 4 \\ -2 & -6 & 4 & -8 \\ 1 & 3 & -1 & 2 \end{bmatrix}$.

1. Find a basis for the column space of A .

$$\begin{bmatrix} 1 & 3 & -2 & 4 \\ -2 & -6 & 4 & -8 \\ 1 & 3 & -1 & 2 \end{bmatrix} \begin{array}{l} R_2 + 2R_1 \rightarrow R_2 \\ R_3 - R_1 \rightarrow R_3 \end{array} \begin{bmatrix} 1 & 3 & -2 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 \end{bmatrix} \begin{array}{l} R_1 + 2R_3 \rightarrow R_1 \\ R_2 \leftrightarrow R_3 \end{array} \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\begin{matrix} \uparrow & \uparrow & & & \uparrow & \uparrow \end{matrix}$

Basis for column space is $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \\ -1 \end{bmatrix} \right\}$

2. Find a basis for the row space of A .

From the above work, basis is

$$\mathcal{B}' = \left\{ (1, 3, 0, 0), (0, 0, 1, -2) \right\}$$

3. $\text{rank}(A) = |\mathcal{B}| = \boxed{2}$

4. $\text{nullity}(A) = n - \text{rank}(A) = 4 - 2 = \boxed{2}$