

Name: Richard

MATH 310

R. Hammack

Score: 10**Directions:** Please answer in the space provided. No calculators. Please put all phones, etc., away.

1. Suppose  $A$ ,  $B$  and  $X$  are invertible matrices, and  $AX^{-1} = B$ . Express  $X$  in terms of  $A$  and  $B$ .

$$AX^{-1} = B$$

$$AX^{-1}X = BX$$

$$AI = BX$$

$$A = BX$$

$$B^{-1}A = B^{-1}BX$$

$$B^{-1}A = IX$$

$$X = B^{-1}A$$

2. Find the inverse of the matrix  $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ , if it exists, or verify that it does not exist.

$$\left[ \begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2} \left[ \begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\frac{1}{2}R_3 \rightarrow R_3}$$

$$\left[ \begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{2} \end{array} \right] \xrightarrow{\begin{array}{l} R_1 - R_2 \rightarrow R_1 \\ R_1 - R_3 \rightarrow R_1 \end{array}} \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & -\frac{1}{2} \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{2} \end{array} \right]$$

Answer:  $A^{-1} = \begin{bmatrix} -1 & 1 & -\frac{1}{2} \\ 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$

Check:  $AA^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 & -\frac{1}{2} \\ 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

