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TEST 1

MATH 200, SECTION 9
March 12, 2021

Directions: Closed book, closed notes, no calculators. Put all phones, etc., away. You will need only a pencil or pen.

1. (10 points) Use a **limit definition** of the derivative to find the derivative of $f(x) = \dfrac{1}{x+1}$.

$$f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x} = \lim_{z \to x} \frac{\frac{1}{z+1} - \frac{1}{x+1}}{z - x}$$

$$= \lim_{z \to x} \frac{\frac{1}{z+1} - \frac{1}{x+1}}{z - x} \cdot \frac{(z+1)(x+1)}{(z+1)(x+1)}$$

$$= \lim_{z \to x} \frac{(x+1) - (z+1)}{(z-x)(z+1)(x+1)}$$

$$= \lim_{z \to x} \frac{x - z}{(z-x)(z+1)(x+1)}$$

$$= \lim_{z \to x} \frac{-(z-x)}{(z-x)(z+1)(x+1)}$$

$$= \lim_{x \to 1} \frac{-1}{(z+1)(x+1)} = \frac{-1}{(x+1)(x+1)} = \boxed{\frac{-1}{(x+1)^2}}$$

2. (10 points) The graph of a function $f(x)$ is sketched below.

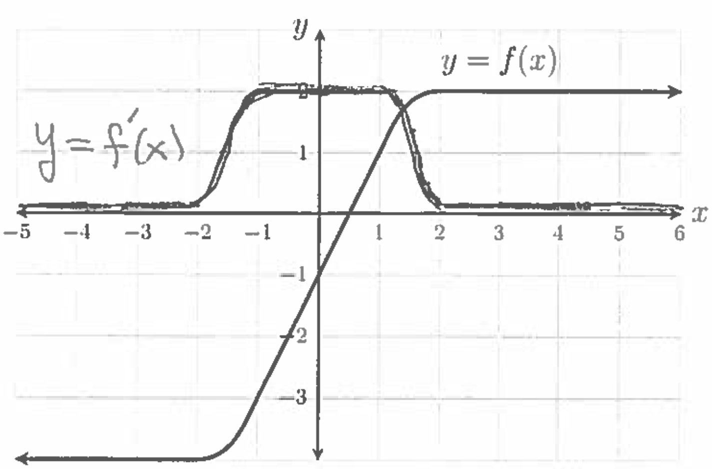
(a) Using the same coordinate axis, sketch a graph of the derivative $f'(x)$.

(b) Suppose $g(x) = (f(x))^4$. Find $g'(1)$.

$$g'(x) = 4(f(x))^3 f'(x) \qquad \text{Thus } g'(1) = 4(f(1))^3 f'(1)$$

$$= 4(1)^3 \cdot 2$$

$$= \boxed{8}$$



3. (48 points) Find the derivatives of these functions. You do **not** need to simplify your answers.

(a) $f(x) = 5x^7 + 3x - \sqrt{2}$

$\boxed{f'(x) = 35x^6 + 3}$

(b) $f(x) = \sin(x) + \sec(x)$

$\boxed{f'(x) = \cos(x) + \sec(x)\tan(x)}$

(c) $f(x) = \sin(x)\sec(x)$

$\boxed{f'(x) = \cos(x)\sec(x) + \sin(x)\sec(x)\tan(x)}$

(d) $f(x) = \sin\big(\sec(x)\big)$

$\boxed{f'(x) = \cos\big(\sec(x)\big)\sec(x)\tan(x)}$

(e) $f(x) = \sec\big(\sin(x)\big)$

$\boxed{f'(x) = \sec\big(\sin(x)\big)\tan\big(\sin(x)\big)\cos(x)}$

(f) $f(x) = \dfrac{\tan(x)}{x^2 + e^x}$

$$\boxed{f'(x) = \frac{\sec^2(x)(x^2+e^x) - \tan(x)(2x+e^x)}{(x^2+e^x)^2}}$$

(g) $f(x) = \sqrt{e^x + x}$

$= (e^x + x)^{\frac{1}{2}}$

$$f'(x) = \frac{1}{2}(e^x + x)^{\frac{1}{2}-1}(e^x + 1)$$

$$= \boxed{\frac{e^x + 1}{2\sqrt{e^x + x}}}$$

(h) $y = \cos\left(e^{x^2+x}\right)$

$$\boxed{f'(x) = -\sin\left(e^{x^2+x}\right)e^{x^2+x}(2x+1)}$$

4. (10 points) Given that $z = \sin(w^2)$, find $\dfrac{d^2z}{dw^2}$.

$$\frac{dz}{dw} = \cos(w^2)\,2w$$

$$\frac{d^2z}{dw^2} = -\sin(w^2)\,2w \cdot 2w + \cos(w^2) \cdot 2$$

$$= \boxed{-4w^2\sin(w^2) + 2\cos(w^2)}$$

5. (10 points) Find the equation of the tangent line to the graph of $f(x) = \sqrt{x}$ at $(4, f(4))$.

$$f(x) = x^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

Slope of tangent line: $m = f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

Point on tangent line: $(4, \sqrt{4}) = (4, 2)$.

By point-slope formula: $y - y_0 = m(x - x_0)$

$$y - 2 = \frac{1}{4}(x - 4)$$

$$\boxed{y = \frac{1}{4}x + 1}$$

6. (10 points) Find all x for which the tangent to the graph of $f(x) = e^{x^3 - 27x}$ at $(x, f(x))$ is horizontal.

Need to solve $f'(x) = 0$

$$e^{x^3-27x}(3x^2-27) = 0$$

$$3e^{x^3-27x}(x^2-9) = 0$$

$$3e^{x^3-27x}(x-3)(x+3) = 0$$

$x = 3$ $\qquad$ $x = -3$

$\boxed{\text{Answer: } x = 3 \text{ and } x = -3}$