

Section 4.3 Graphing Functions

This section is about using the information that $f'(x)$ and $f''(x)$ give to draw careful and accurate graphs of $f(x)$. Recall the following:

Increase/Decrease

- If $f'(x) > 0$ on (a, b) then $f(x)$ increases on (a, b) .
- If $f'(x) < 0$ on (a, b) then $f(x)$ decreases on (a, b) .

Concavity

- If $f''(x) > 0$ on (a, b) then $f(x)$ is concave up on (a, b) .
- If $f''(x) < 0$ on (a, b) then $f(x)$ is concave down on (a, b) .

Example Sketch graph of $f(x) = \ln(x^2 + 1)$

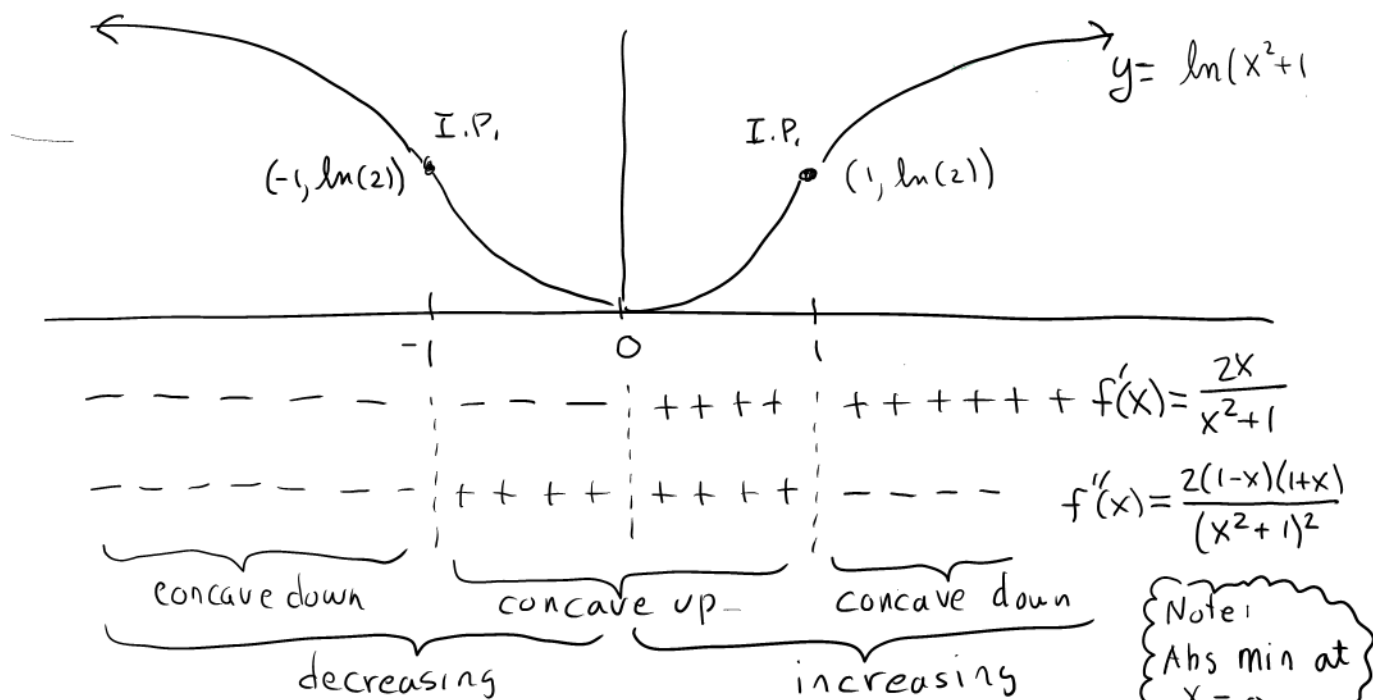
Increasing/decreasing

$$f'(x) = \frac{2x}{x^2 + 1}$$

positive when x positive
negative when x negative

Concavity

$$f''(x) = \frac{2(x^2 + 1) - 2x \cdot 2x}{(x^2 + 1)^2} = \frac{2x^2 + 2 - 4x^2}{(x^2 + 1)^2} = \frac{2(1 - x^2)}{(x^2 + 1)^2} = \frac{2(1 - x)(1 + x)}{(x^2 + 1)^2}$$



Note:
Abs min at
 $x = 0$