Textbook Briggs and Cochran

Part 2 Limits

Today we'll begen discussing a new concept: Limits.
This is what separates algebra from calculus:

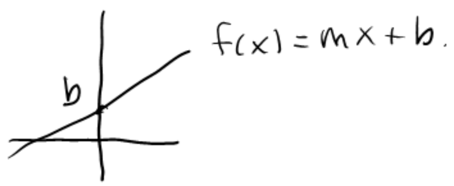
$$\begin{pmatrix}\text{algebra}\\ \text{\& trig}\end{pmatrix} + (\text{limits}) = \text{CALCULUS}.$$

Calculus is buit on the foundation of algebra and functions
Up until now we have been reviewing algebra & functions.
We can now get to the main, simple idea behind all of calculus.
It centers on functions

Functions can be put in to two groups - linear & non-linear

Linear Functions

$f(x) = mx + b$



These functions are very simple. The graph of $f(x) = mx + b$ is a line with slope m and y-intercept b

Non-Linear Functions

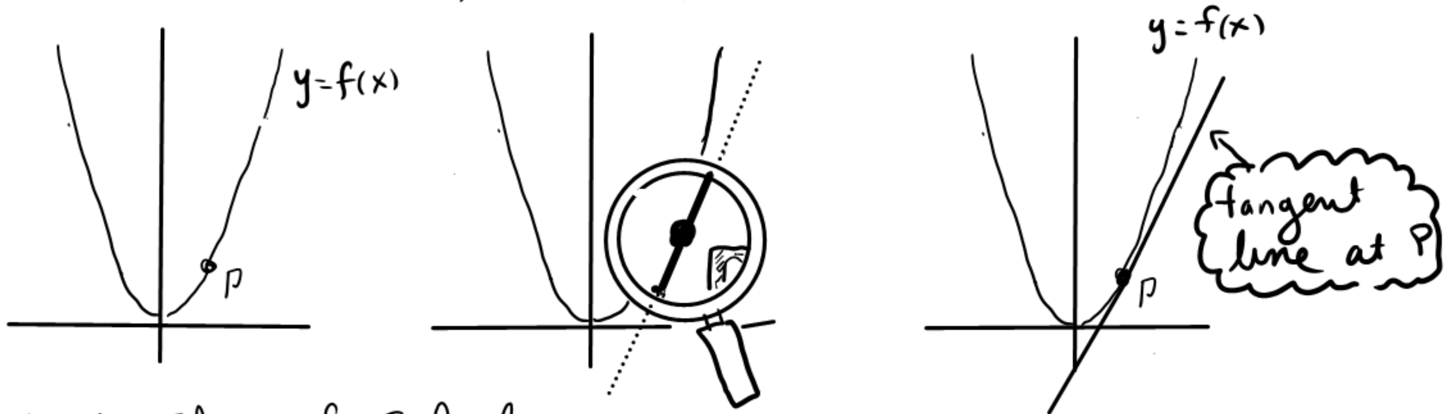
Anything else: $f(x) = x^2$
$f(x) = \sin(x)$ $f(x) = \tan^{-1}(x)$ etc.

graph is not a straight line

Non-linear functions can be quite complicated.

Dilemma Linear functions are simple, but most useful functions are non-linear.

The central idea of calculus addresses this. Consider a non-linear function, and a point P on it.

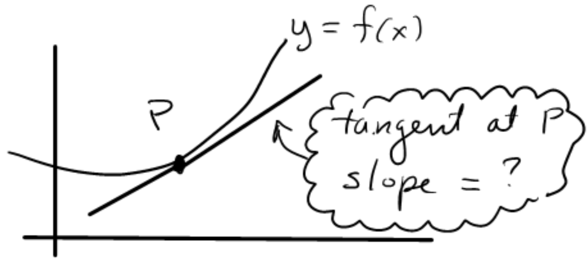


Central Idea of Calculus

Up close, non-linear functions look linear.

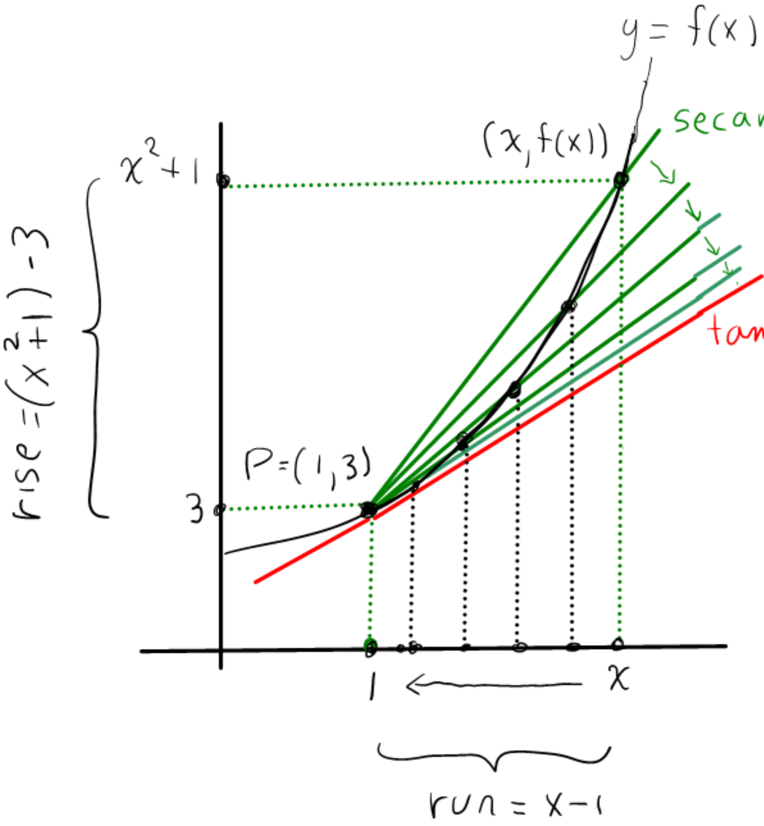
Sections 2.1, 2.2 Introduction to limits

One of the main objectives of calculus is to determine the slope of a tangent line to the graph of $f(x)$



Today we'll see how this question can be answered with a new concept called a limit. This concept separates calculus from algebra.

Motivational Problem Find the slope of the line tangent to graph of $f(x) = x^2 + 2$ at the point $P = (1,3)$.



secant $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{(x^2+2-3)}{x-1} = \dfrac{x^2-1}{x-1} = \dfrac{(x+1)(x-1)}{x-1} = x+1$

provided $x \neq 1$

As x approaches 1, secant rotates towards tangent so secant slope approaches tangent slope i.e. $\dfrac{x^2-1}{x-1}$ approaches tangent slope.

x	secant slope $= \dfrac{x^2-1}{x-1} = x+1$
2	3
1.5	2.5
1.1	2.1
1.01	2.01
1.001	2.001
1.0001	2.0001
$\downarrow$	$\downarrow$
1	2

It appears that tangent slope = 2

We express this situation as

$$\text{tangent slope} = \lim_{x \to 1} \frac{x^2-1}{x-1} = 2$$

tangent slope $= \lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1}$

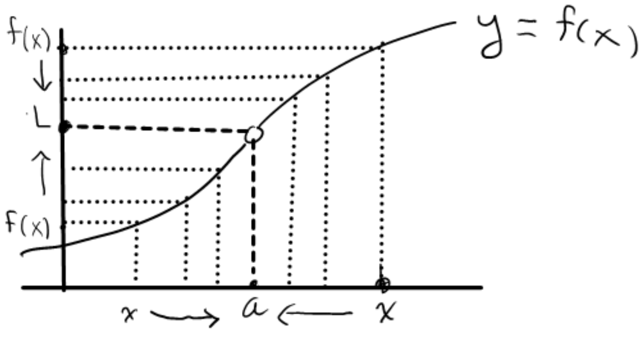
$\lim\limits_{x \to a} g(x)$, $\lim\limits_{x \to a} f(x)$ In general, limits have this form

So we see that in order to compute tangent slopes, we must understand expressions like $\lim\limits_{x \to a} g(x)$ or $\lim\limits_{x \to a} f(x)$.

Plan: Study limits in Part 2 of text, then return to finding tangent slopes in Part 3

Definition $\lim\limits_{x \to a} f(x)$ is the value that $f(x)$ approaches as x approaches a.

In this picture $\lim\limits_{x \to a} f(x) = L$

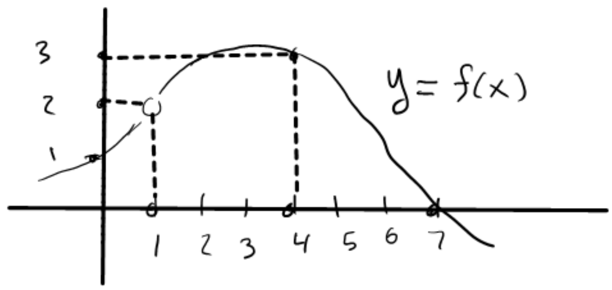


Example

$\lim\limits_{x \to 1} f(x) = 2$

$\lim\limits_{x \to 4} f(x) = 3$

$\lim\limits_{x \to 7} f(x) = 0$



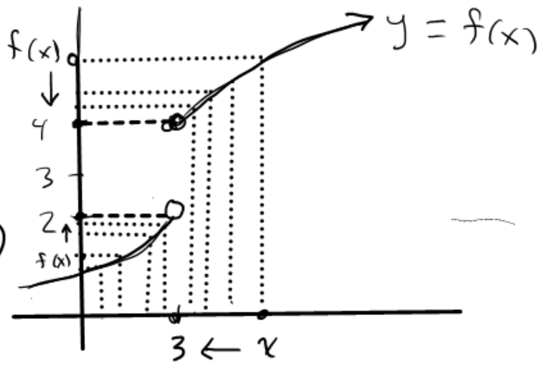
One-sided limits

Right-hand limit: $\lim\limits_{x \to 3^+} f(x) = 4$

x approaches 3 from right

Left-hand limit: $\lim\limits_{x \to 3^-} f(x) = 2$

x approaches 3 from left



Regular limit $\lim\limits_{x \to 3} f(x)$ Does not exist (DNE)

Theorem $\lim\limits_{x \to a} f(x) = L \iff \lim\limits_{x \to a^+} f(x) = L = \lim\limits_{x \to a^-} f(x)$