MATH 200 Calculus I (Section 7)

Richard Hammack

Today • Function Review, Trig Review

Wed. • Trig Review

Thursday • Review of Inverse functions & logarithms

You should get a copy of the textbook within the next week.

But we will work from handouts for the first four class meetings.

Assignment #1 **Due Thursday January 21**

From Hammacks' Trig Review
(See link on Jan 19 entry of course calendar)

Section 3.1: 2 8 14 20 22
Section 3.2: 7, 12
Section 3.4: 2

extra point for each typo found in my handout

CALCULUS PRELIMINARIES

In any branch of science there are situations in which one variable quantity depends on another

- Gravitational force depends on distance between objects
- Force on an object depends on its acceleration
- Bond prices depend on interest rates.

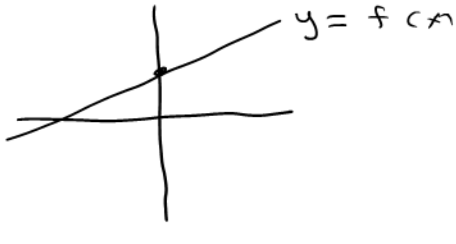
Function: A mathematical construction that models how one quantity depends on another.

$$\underset{\text{(output)}}{\text{dependent variable}} \rightarrow y = f(x) \leftarrow \underset{\text{(input)}}{\text{independent variable}}$$

Calculus is built on the notion of functions. We will spend a week reviewing functions before getting to calculus. Trig functions, inverse functions, exponential functions, logarithms, inverse trig functions

Recall

The domain of $f(x)$ is the set of all possible input values x.
The range of $f(x)$ is the set of all possible output values $y=f(x)$



Example $f(x) = \frac{1}{2}x + 1$.

domain: $\mathbb{R} = (-\infty, \infty)$ (all real #'s)

range $\mathbb{R} = (-\infty, \infty)$

Example $f(x) = $ (shaded area)

$$f(x) = \underbrace{\tfrac{1}{2}x}_{\text{triangle}} + \underbrace{1}_{\text{square}}$$

domain $\{x \mid x \geq 0\} = [0, \infty)$ all pos. x values

range $\{y \mid y \geq 1\} = [1, \infty)$

Note In these examples the function is $f(x) = \frac{1}{2}x + 1$, but its interpretations differ. Consequently the domains and ranges differ too, accordingly.

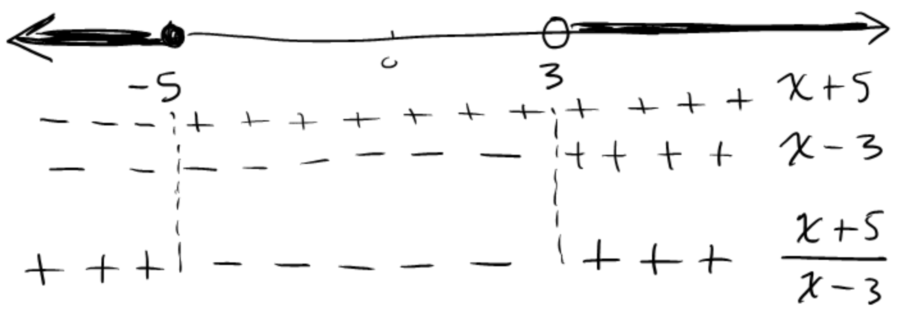
If not stated or implied otherwise, the domain of a function $f(x)$ is assumed to be the set of all input values x for which the function is defined.

Ex $f(x) = \sqrt{x}$ Domain $[0, \infty)$

Ex $f(x) = \dfrac{1}{x}$ Domain $(-\infty, 0) \cup (0, \infty)$

Ex $g(x) = \sqrt{\dfrac{x+5}{x-3}}$

Domain will be all x for which $\dfrac{x+5}{x-3} \geq 0$.



Domain of $g(x)$: $\boxed{(-\infty, -5] \cup (3, \infty)}$

(Note $x = 3$ not in domain because $g(3)$ undefined)

In this class you will often have to think out the domain of a functions.

Range of functions is less important, but still should not be ignored

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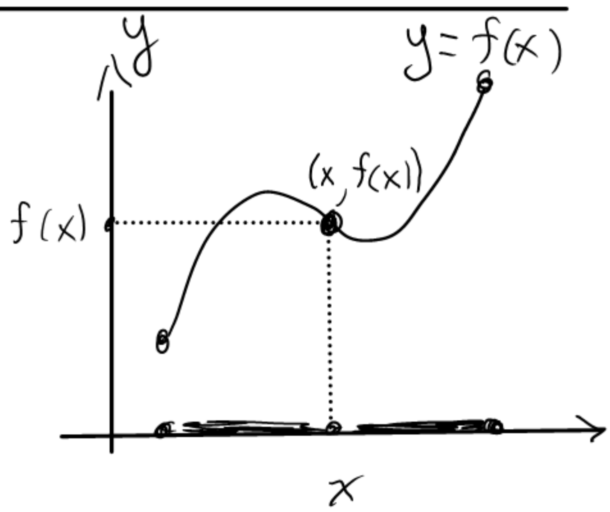
Graph of a function $f(x)$ is the set of all points $(x, f(x))$ for which x is in the domain of f



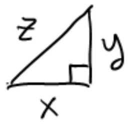
Review of Trig Functions

Trigonometric functions relate angles of right triangles to their sides

Input Angle measurement θ
Output Side length

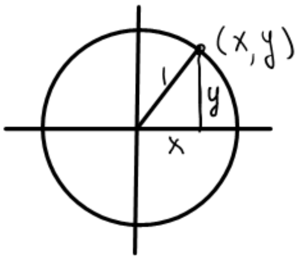
Trig functions are very simple. They have two ingredients:

(A) Pythagorean Theorem $x^2 + y^2 = z^2$

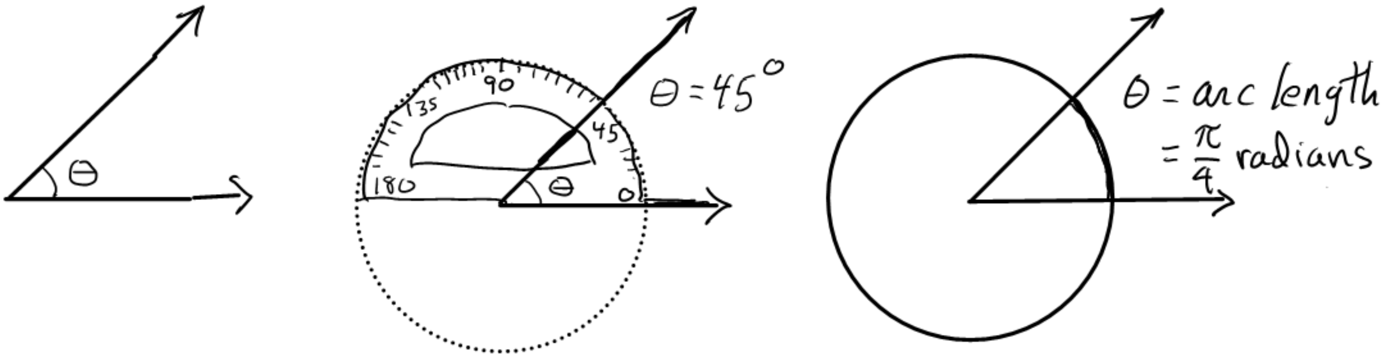


(B) Unit Circle

Graph of $x^2 + y^2 = 1$ is a circle of radius 1 centered at the origin



The unit circle is a protractor for measuring angles.
It measures them in arc length (radians), not degrees.

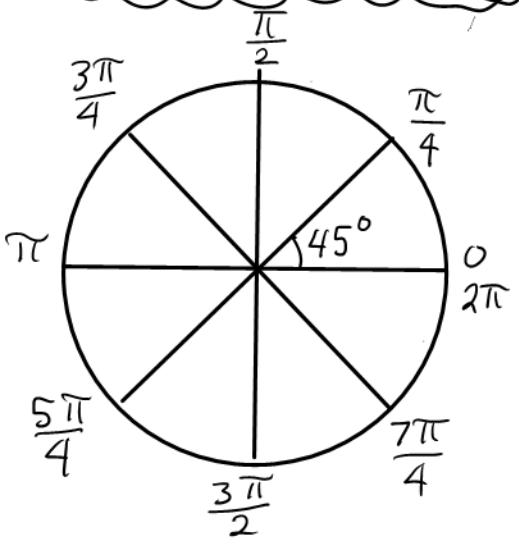


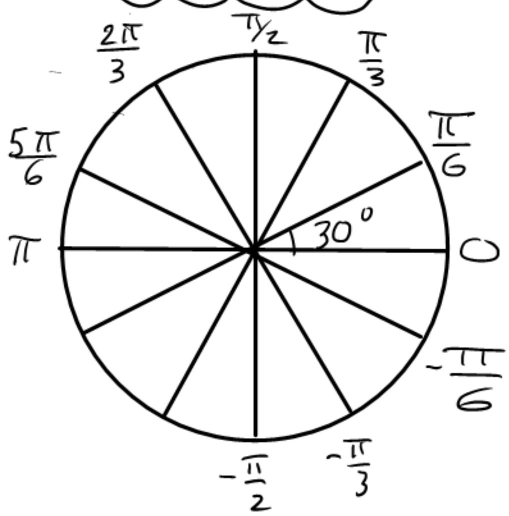
Measure θ ... with protractor, $\theta = 45^\circ$ ... with unit circle $\theta = \frac{\pi}{4}$ radians

man-made
artificial, arbitrary

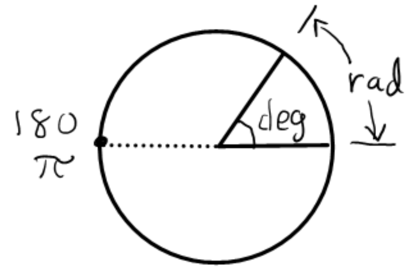
Made by God?
Natural, universal

Common Angles:





Conversions



$$\boxed{\frac{deg}{180} = \frac{rad}{\pi}}$$

$$\boxed{deg = \frac{rad}{\pi}180}$$

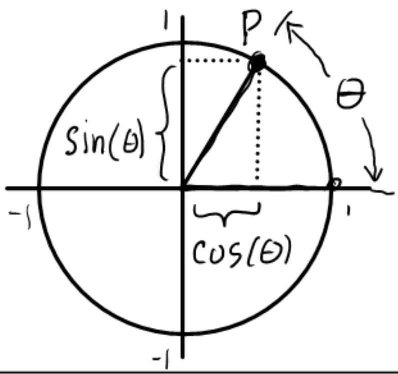
$$\boxed{rad = \frac{deg}{180}\pi}$$

Example: $40^\circ = \frac{40}{180}\pi = \frac{2}{9}\pi$ radians

Definitions

sin and cos are functions of radian measure

$\cos(\theta) =$ x-coordinate of P
$\sin(\theta) =$ y-coordinate of P

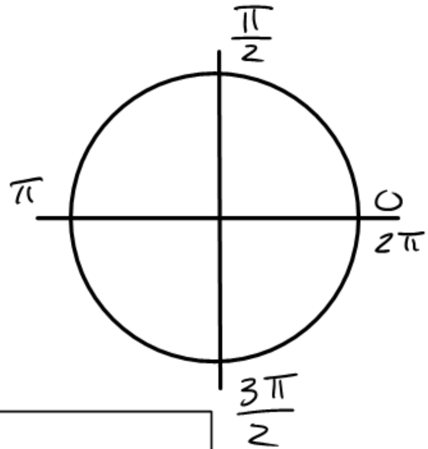


Examples

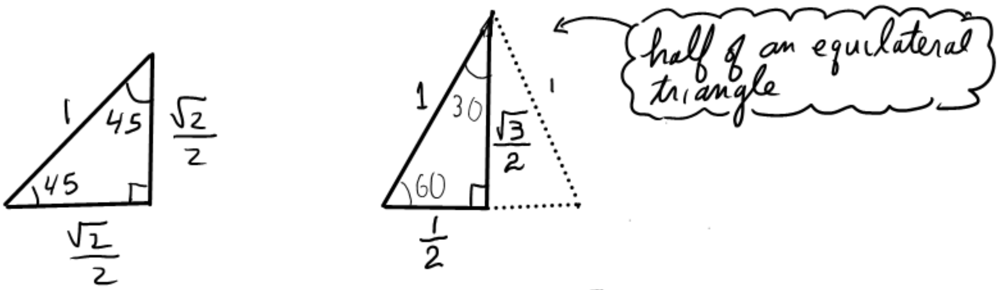
$\cos(0) = 1$ $\quad \cos(\frac{\pi}{2}) = 0$ $\quad \cos(\pi) = -1$

$\sin(0) = 0$ $\quad \sin(\pi/2) = 1$ $\quad \sin(\pi) = 0$

$\cos(2\pi) = 0$ $\quad \sin(\frac{7\pi}{2}) = -1$



For other angles, helpful to know these triangles:

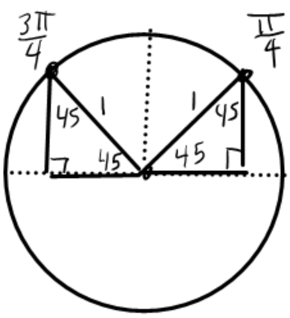


$\cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$\sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$\cos(\frac{3\pi}{4}) = -\frac{\sqrt{2}}{2}$

$\sin(\frac{3\pi}{4}) = \frac{\sqrt{2}}{2}$

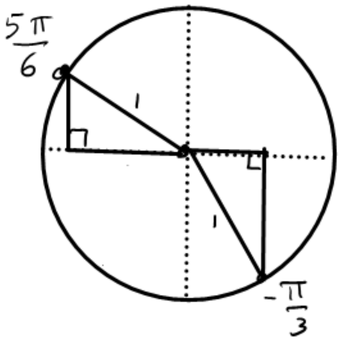


$\cos(-\frac{\pi}{3}) = \frac{1}{2}$

$\sin(-\frac{\pi}{3}) = -\frac{\sqrt{3}}{2}$

$\cos(\frac{5\pi}{6}) = -\frac{\sqrt{3}}{2}$

$\sin(\frac{5\pi}{6}) = \frac{1}{2}$



The six trig functions

$\sin(\theta)$

$\cos(\theta)$

$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$

$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$

$\sec(\theta) = \frac{1}{\cos(\theta)}$

$\csc(\theta) = \frac{1}{\sin(\theta)}$

Examples

$$\tan\left(\frac{\pi}{4}\right) = \frac{\sin(\pi/4)}{\cos(\pi/4)} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \boxed{1}$$

$$\sec\left(\frac{\pi}{4}\right) = \frac{1}{\cos(\pi/4)} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\sqrt{2}}$$

Notation

- OK to write $\cos\theta$ instead of $\cos(\theta)$
(Even though like $f x$ instead of $f(x)$!)

- Abbreviations: $(\cos(\theta))^2 = \cos^2(\theta) = \cos^2\theta$
$(\tan(\theta))^3 = \tan^3(\theta) = \tan^3\theta$ etc.

- Any independent variable is OK.

$\sin(\theta)$ $\quad \sin(x)$ $\quad \sin(t)$ $\quad \sin\left(\frac{x^2+3x+1}{2}\right)$