§ 3.1

(32) $f(x) = \frac{1}{x^2}$ $\qquad a = 1$

(a) $$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{\frac{1}{(1+h)^2} - \frac{1}{1^2}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{(1+h)^2} - 1}{h} \cdot \frac{(1+h)^2}{(1+h)^2} = \lim_{h \to 0} \frac{1 - (1+h)^2}{h(1+h)^2}$$

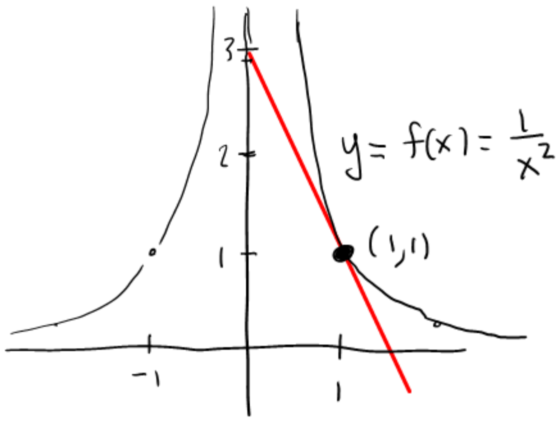
$$= \lim_{h \to 0} \frac{1 - 1 - 2h - h^2}{h(1+h)^2} = \lim_{h \to 0} \frac{h(-2-h)}{h(1+h)^2}$$

$$= \lim_{h \to 0} \frac{-2-h}{(1+h)^2} = \frac{-2+0}{(1+0)^2} = \boxed{-2}$$

(b) Find the equation of the tangent to $y = f(x) = \frac{1}{x^2}$ at The point $(1, f(1)) = 1-1.$

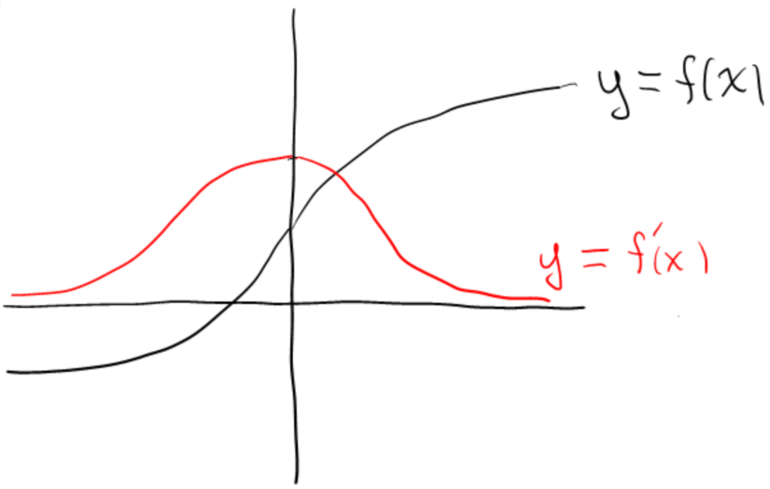
By point-slope formula,

$$y - y_0 = m(x - x_0)$$
$$y - 1 = f'(1)(x - 1)$$
$$y - 1 = -2(x - 1)$$
$$\boxed{y = -2x + 3}$$



§ 3.2

(12)



§ 3.3

(10) $g(x) = e^3$, $\boxed{g'(x) = 0}$ (because e^3 is a constant)

(18) $f(s) = \frac{\sqrt{s}}{4} = \frac{1}{4} s^{\frac{1}{2}}$ $\quad f'(s) = \frac{1}{4}\frac{1}{2} s^{\frac{1}{2}-1} = \frac{1}{8} s^{-\frac{1}{2}} = \boxed{\frac{1}{8\sqrt{s}}}$

(36)(a) $y = f(x) = x^3 - 4x^2 + 2x - 1$

$f'(x) = 3x^2 - 8x + 2$.

Thus slope of tangent to graph at $(2, f(2)) = (2, -5)$ is $m = f'(2) = 3 \cdot 2^2 - 8 \cdot 2 + 2 = -2$

By point-slope formula, tangent has equation

$$y - y_0 = m(x - x_0)$$

$$y - (-5) = -2(x-2)$$

$$y = -2x + 4 - 5$$

$$\boxed{y = -2x - 1}$$

(40) $f(t) = t^3 - 27t + 5$

$f'(t) = 3t^2 - 27$

(a) Slope of $y = f(t)$ is 0 when $f'(t) = 0$

$$3t^2 - 27 = 0$$

$$3(t+3)(t-3) = 0$$

$\boxed{\text{Thus slope is zero when } t = 3 \text{ or } t = -3}$

(b) Slope of $y = f(t)$ is 21 when $f'(t) = 21$

$$3t^2 - 27 = 21$$

$$3t^2 = 48$$

$$t^2 = 16$$

$$t = \pm 4.$$

$\boxed{\text{Thus slope is 21, when } t = \pm 4}$

(44) $f(x) = 3x^3 + 5x^2 + 6x$

$f'(x) = 9x^2 + 10x + 6$

$f''(x) = 18x + 10$

$f'''(x) = 18$

§ 3.4

(10) $g(w) = e^w(5w^2+3w+1)$

$$g'(w) = \frac{d}{dw}\left[e^w\right](5w^2+3w+1) + e^w \frac{d}{dw}\left[5w^2+3w+1\right]$$
$$= e^w(5w^2+3w+1) + e^w(10w+3)$$
$$= \boxed{e^w(5w^2+13w+4)}$$

(12) $f(x) = \left(1+\frac{1}{x^2}\right)(x^2+1) = (1+x^{-2})(x^2+1)$

$$f'(x) = \frac{d}{dx}\left[1+x^{-2}\right](x^2+1) + (1+x^{-2})\frac{d}{dx}\left[x^2+1\right]$$
$$= (0-2x^{-3})(x^2+1) + (1+x^{-2})(2x+0)$$
$$= -2x^{-1} - 2x^{-3} + 2x + 2x^{-1} = \boxed{2x - \frac{2}{x^3}}$$

(20) $f(x) = \dfrac{x^3-4x^2+x}{x-2}$

$$f'(x) = \frac{(3x^2-8x+1)(x-2) - (x^3-4x^2+x)(1)}{(x-2)^2}$$
$$= \frac{3x^3-8x^2+x-6x^2+16x-2-x^3+4x^2-x}{(x-2)^2}$$
$$= \frac{2x^3-10x^2+16x-2}{(x-2)^2}$$

(28) $y = (2\sqrt{x}-1)(4x+1)^{-1} = \dfrac{2x^{1/2}-1}{4x+1}$

$$\frac{dy}{dx} = \frac{(2\cdot\frac{1}{2}x^{-1/2}+0)(4x+1) - (2x^{1/2}-1)(4+0)}{(4x+1)^2} = \frac{4x^{\frac{1}{2}} + x^{-\frac{1}{2}} - 8x^{\frac{1}{2}} + 4}{(4x+1)^2}$$
$$= \frac{-4x^{\frac{1}{2}} + x^{-\frac{1}{2}} + 4}{(4x+1)^2} = \boxed{\frac{-4\sqrt{x} + \frac{1}{\sqrt{x}} + 4}{(4x+1)^2}} = \frac{\frac{-4x+1+4\sqrt{x}}{\sqrt{x}}}{(4x+1)^2}$$
$$= \boxed{\frac{1-4x+4\sqrt{x}}{\sqrt{x}(4x+1)^2}}$$

(48) $f(x) = (1-2x)e^{-x} = \dfrac{1-2x}{e^x}$

$$f'(x) = \frac{(0-2)e^x - (1-2x)e^x}{(e^x)^2} = \frac{e^x(-2-(1-2x))}{e^x e^x}$$
$$= \boxed{\frac{2x-3}{e^x}}$$