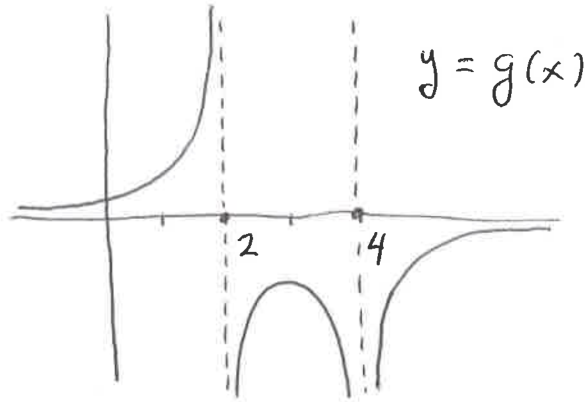
§2.4




(10) (a) $\lim_{x \to 2^-} g(x) = \boxed{\infty}$

(b) $\lim_{x \to 2^+} g(x) = \boxed{-\infty}$

(c) $\lim_{x \to 2} g(x)$ $\boxed{\text{DNE}}$

(d) $\lim_{x \to 4^-} g(x) = \boxed{-\infty}$ (e) $\lim_{x \to 4^+} g(x) = \boxed{-\infty}$ (f) $\lim_{x \to 4} g(x) = \boxed{-\infty}$

(20) (a) $\lim_{x \to 1^+} \dfrac{x-2}{(x-1)^3} = \boxed{-\infty}$

negative, close to 1

positive, close to 0

(b) $\lim_{x \to 1^-} \dfrac{x-2}{(x-1)^3} = \boxed{\infty}$

negative, close to 1

negative, close to 0

(c) $\lim_{x \to 1} \dfrac{x-2}{(x-1)^3}$ $\boxed{\text{DNE}}$

provided $x \neq -7$

(30) $f(x) = \dfrac{x+7}{x^4 - 49x^2} = \dfrac{x+7}{x^2(x^2-49)} = \dfrac{x+7}{x^2(x+7)(x-7)} = \dfrac{1}{x^2(x-7)}$

(a) $\lim_{x \to 7^-} f(x) = \lim_{x \to 7^-} \dfrac{1}{x^2(x-7)} = \boxed{-\infty}$

approaching 0, neg

no vertical asymptote at $x = -7$

(b) $\lim_{x \to 7^+} f(x) = \lim_{x \to 7^+} \dfrac{1}{x^2(x-7)} = \boxed{+\infty}$

approaching 0, pos

(c) $\lim_{x \to -7} f(x) = \lim_{x \to -7} \dfrac{1}{x^2(x-7)} = \dfrac{1}{7^2(-7-7)} = \boxed{-\dfrac{1}{686}}$

(d) $\lim_{x \to 0} f(x) = \lim_{x \to 0} \dfrac{1}{x^2(x-7)} = \boxed{-\infty}$

approaching 0, negative

From above, $\boxed{\text{lines } x = 7 \text{ and } x = 0 \text{ are vertical asymptotes}}$

§ 2.5

(14) $\lim_{x\to-\infty}\left(5+\frac{100}{x}+\frac{\sin^4(x^3)}{x^2}\right) = 5+0+0 = \boxed{5}$

(32) $\lim_{x\to\infty}\frac{12x^8-3}{3x^8-2x^7} = \lim_{x\to\infty}\frac{12x^8-3}{3x^8-2x^7}\frac{\frac{1}{x^8}}{\frac{1}{x^8}} = \lim_{x\to\infty}\frac{12-\frac{3}{x^8}}{3-\frac{2}{x}} = \frac{12-0}{3-0} = \boxed{4}$

$\lim_{x\to-\infty}\frac{12x^8-3}{3x^8-2x^7} = \boxed{4}$ (by the same process)

Therefore the $\boxed{\text{line } y=4 \text{ is a horizontal asymptote}}$

(54) $f(x) = \frac{\sqrt{16x^4+64x^2}+x^2}{2x^2-4} = \frac{\sqrt{16x^4+64x^2}+x^2}{2(x-\sqrt{2})(x+\sqrt{2})}$ ← factored form

- $\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\frac{\sqrt{16x^4+64x^2}+x^2}{2x^2-4}\frac{\frac{1}{x^2}}{\frac{1}{x^2}}$

$= \lim_{x\to\infty}\frac{\sqrt{16x^4+64x^2}\,\frac{1}{x^2}+1}{2-\frac{4}{x^2}} = \lim_{x\to\infty}\frac{\sqrt{(16x^4+64x^2)\frac{1}{x^4}}+1}{2-\frac{4}{x^2}}$

$= \lim_{x\to 0}\frac{\sqrt{16+\frac{64}{x^2}}+1}{2-\frac{4}{x^2}} = \frac{\sqrt{16+0}+1}{2-0} = \frac{4+1}{2} = \boxed{\frac{5}{2}}$

- Also $\lim_{x\to-\infty} f(x) = \boxed{5/2}$ by the same process.

Thus $\boxed{\text{line } y=\frac{5}{2} \text{ is a horizontal asymptote}}$

- Looking at the factored form (above) we see that the denominator of $f(x)$ is zero for $x=\sqrt{2}$, $x=-\sqrt{2}$. These are the candidates for the locations of the vertical asymptotes. Notice that

$\lim_{x\to\sqrt{2}^+} f(x) = \infty$ and $\lim_{x\to-\sqrt{2}^+} f(x) = -\infty$ because in each case the denominator approaches 0 while the numerator approaches a positive number.

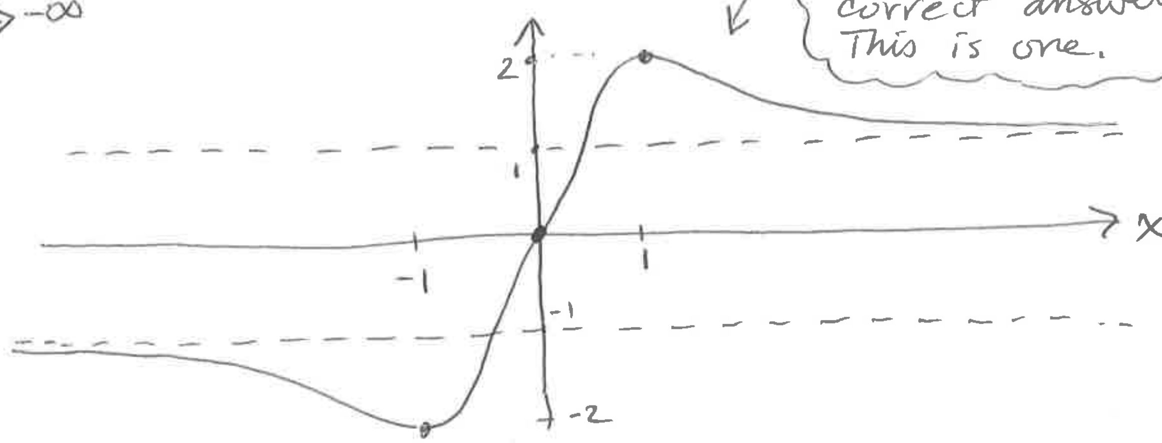
Thus $\boxed{\text{lines } x=\sqrt{2} \text{ and } x=-\sqrt{2} \text{ are vertical asymptotes}}$

§2.5

(66) $f(-1) = -2$, $f(1) = 2$, $f(0) = 0$, $\lim_{x\to\infty} f(x) = 1$,

$\lim_{x\to -\infty} f(x) = -1$

There are *many* correct answers. This is one.




(68) Find the vertical and horizontal asymptotes of $f(x) = e^{1/x}$

Horizontal asymptotes:

$$\lim_{x\to\infty} e^{1/x} = e^{\lim_{x\to\infty} \frac{1}{x}} = e^0 = 1$$

$$\lim_{x\to -\infty} e^{1/x} = e^{\lim_{x\to -\infty} \frac{1}{x}} = e^0 = 1$$

line $y = 1$ is horizontal asymptote

Vertical asymptotes will be those lines $x = a$ for which $\lim_{x\to a^\pm} f(x) = \pm\infty$. If $a \neq 0$, then $\lim_{x\to a^\pm} f(x) = \lim_{x\to a} e^{1/x} = e^{1/a} \neq \infty$ Thus $a = 0$ is the only possible location for a vertical asymptote. Checking:

$\lim_{x\to 0^+} e^{1/x} = \infty$ (because $\frac{1}{x} \to +\infty$)

line $x = 0$ is vertical asymptote

$\lim_{x\to 0^-} e^{1/x} = 0$ (because $\frac{1}{x} \to -\infty$)