

# **MATH 123**

## (Day 4)

### The Fourth Dimension and Cubism

Richard Hammack

<http://www.people.vcu.edu/~rhammack/Math123/>

## **References:**

Linda Henderson

*The Fourth Dimension and Non-Euclidean Geometry in Modern Art*  
Princeton University Press, 1983

Tony Robbin

*Shadows of Reality: The Fourth Dimension in Relativity, Cubism and Modern Thought*

Yale University Press, 2006

## Pioneers of 4-D Geometry

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Georg F.B. Riemann  
1826–1866

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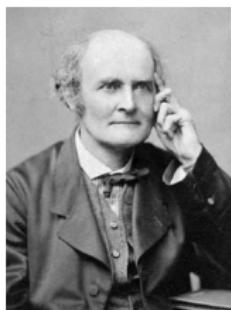
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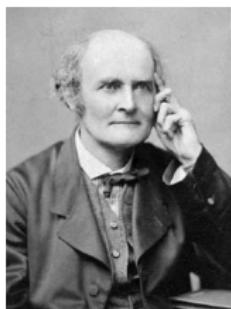
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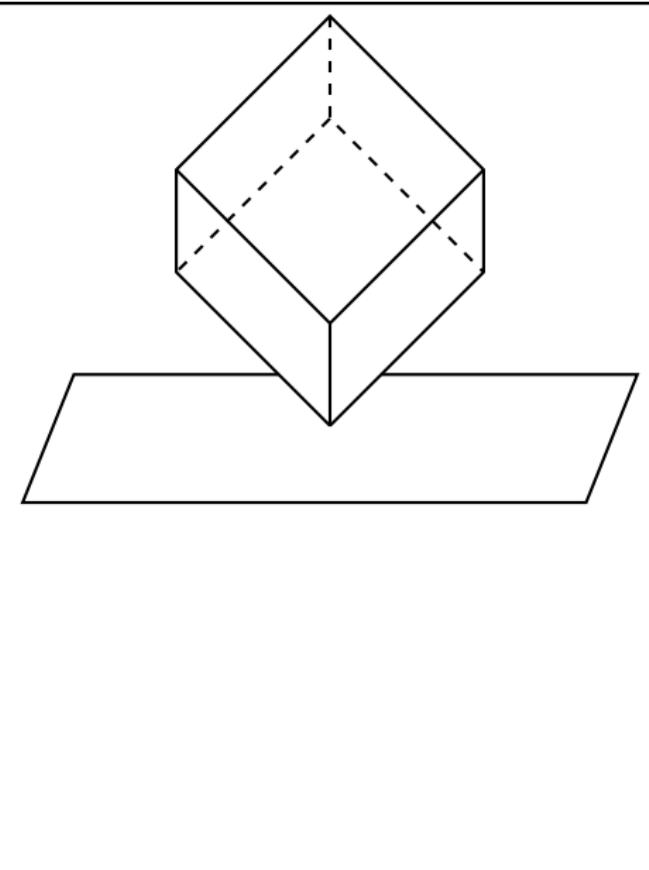


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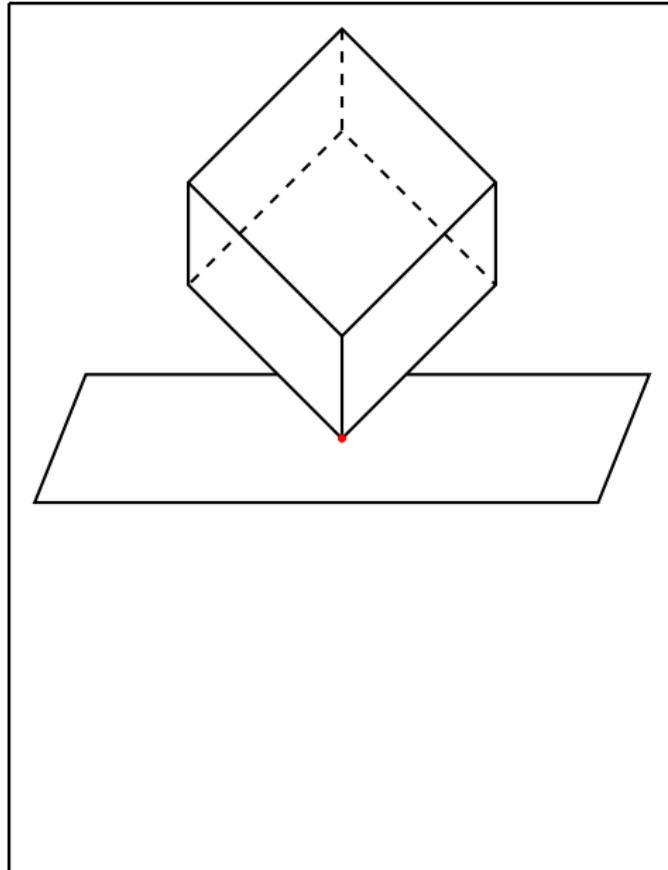


Alicia Boole Stott  
1860–1940

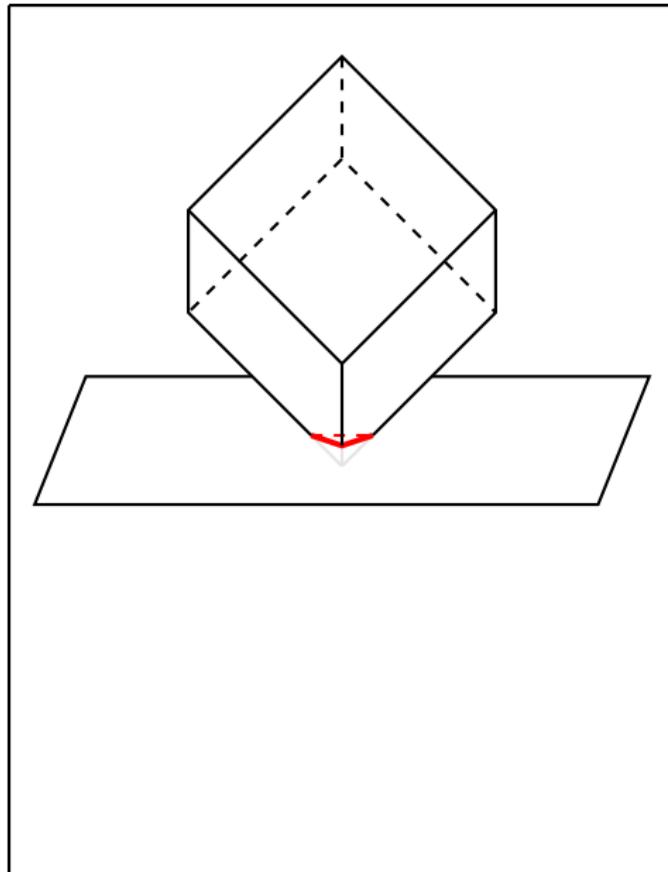
## 2-D Cross-sections of a cube



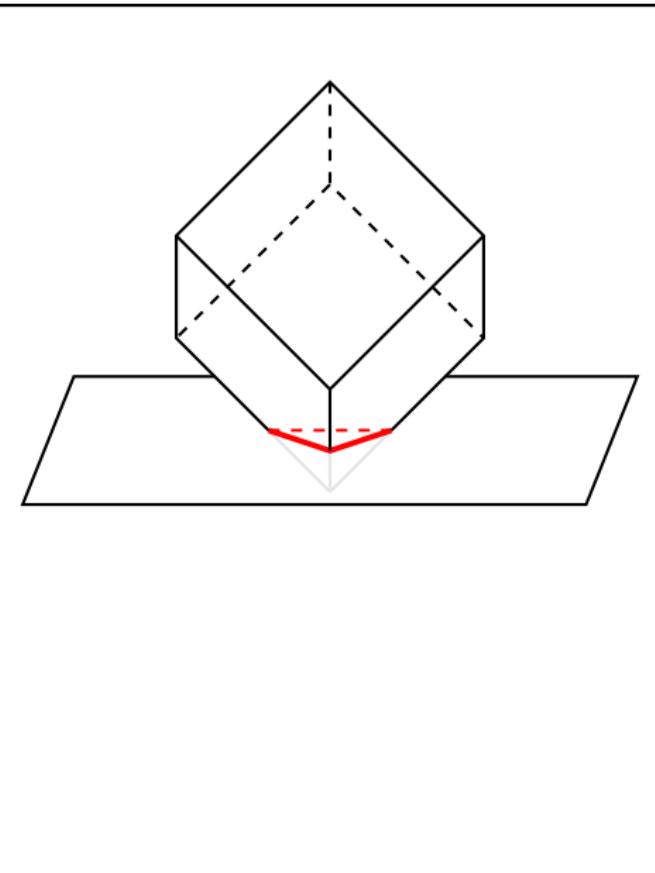
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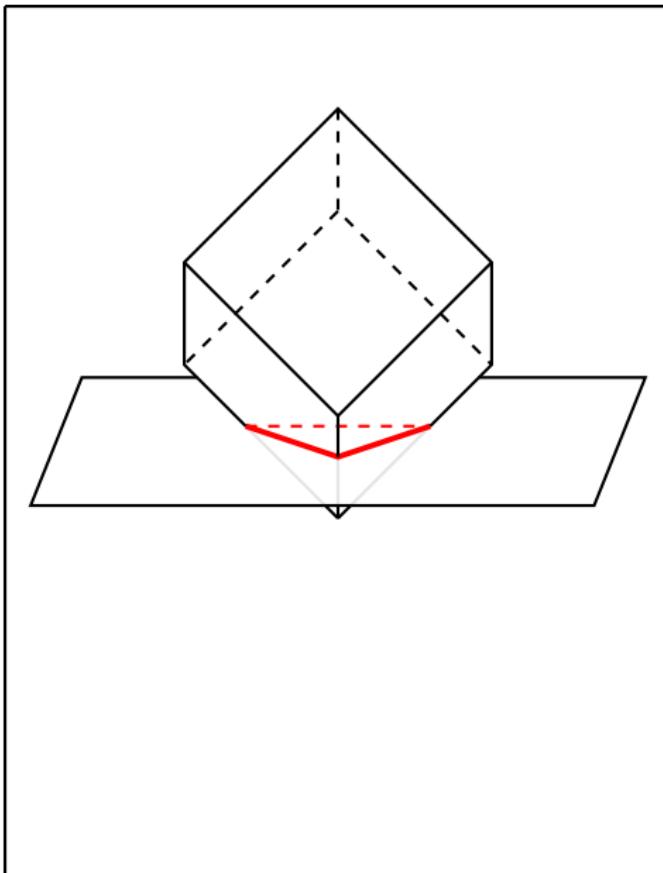
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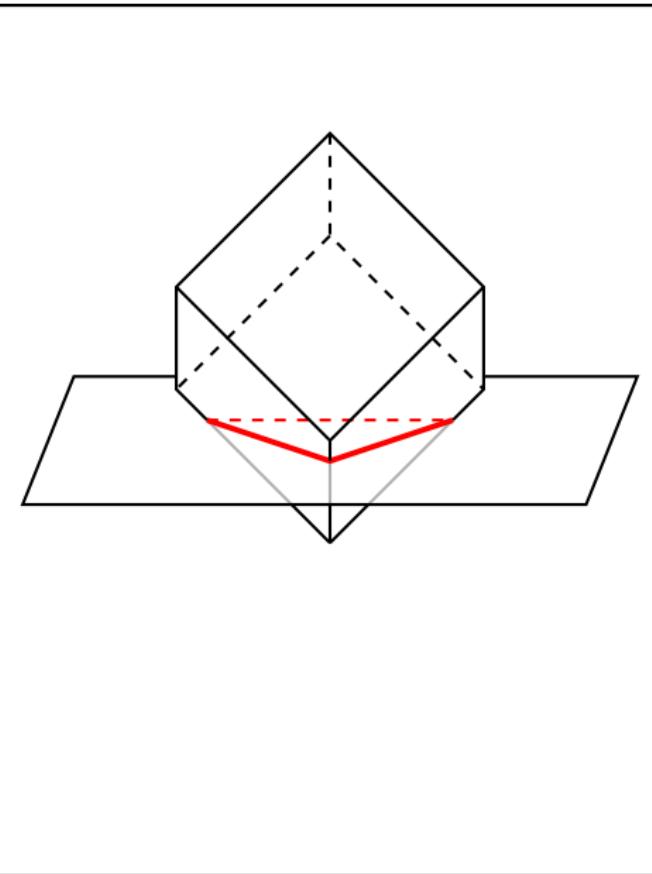
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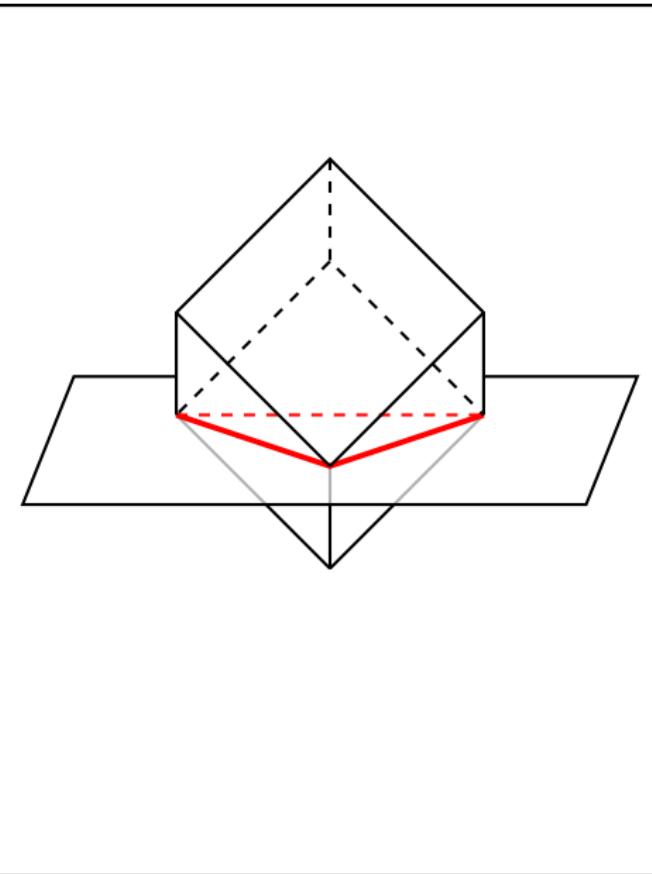
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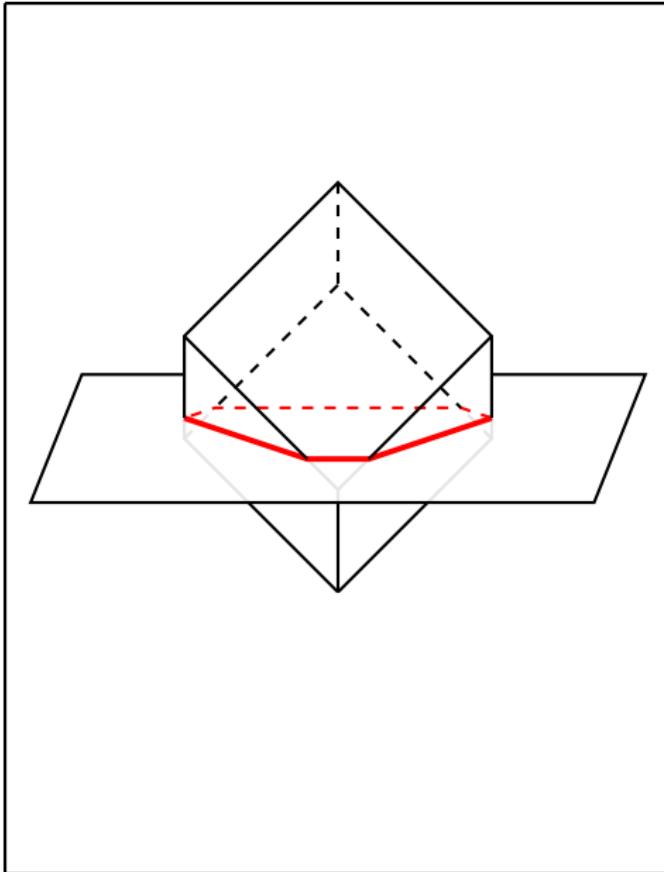
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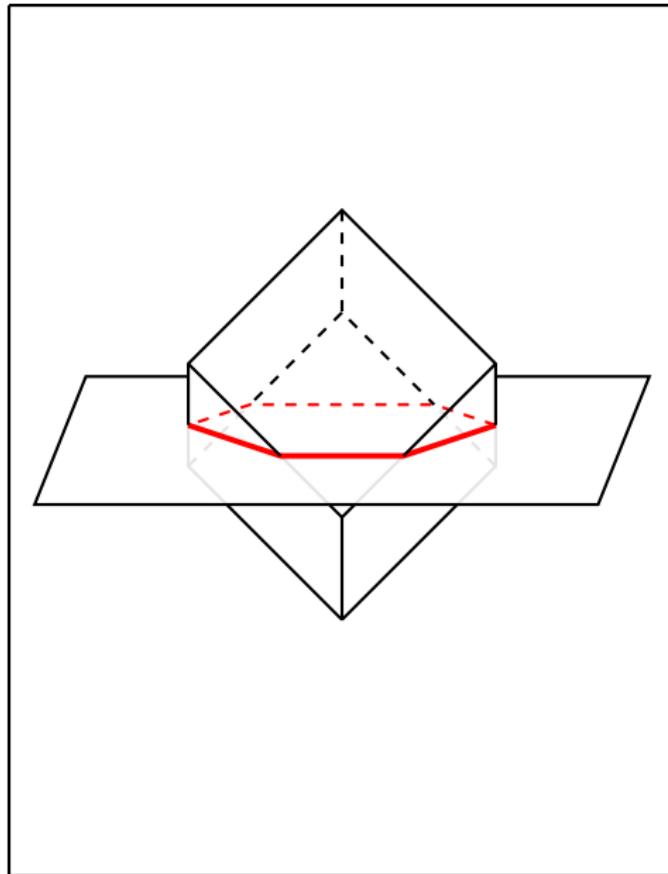
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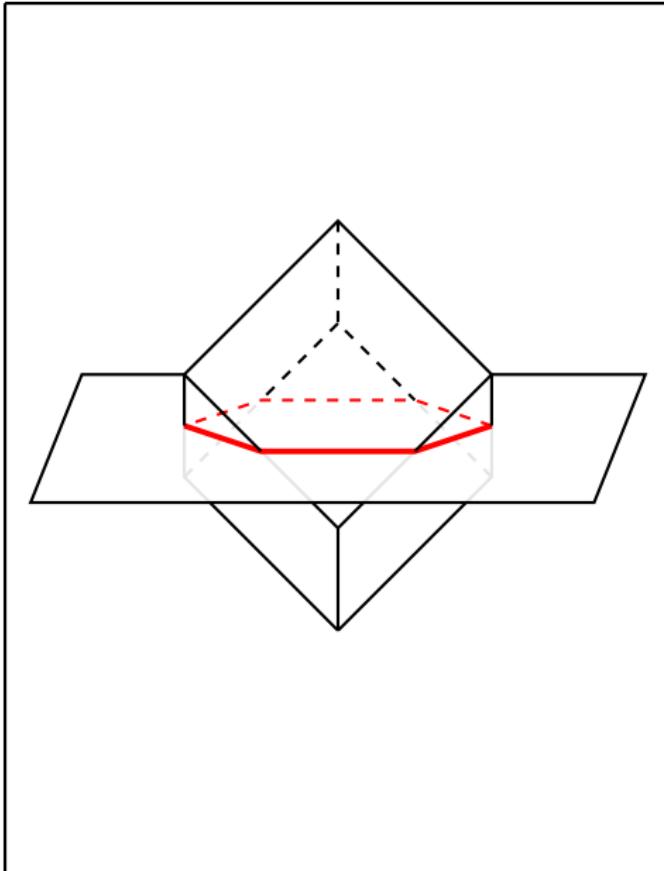
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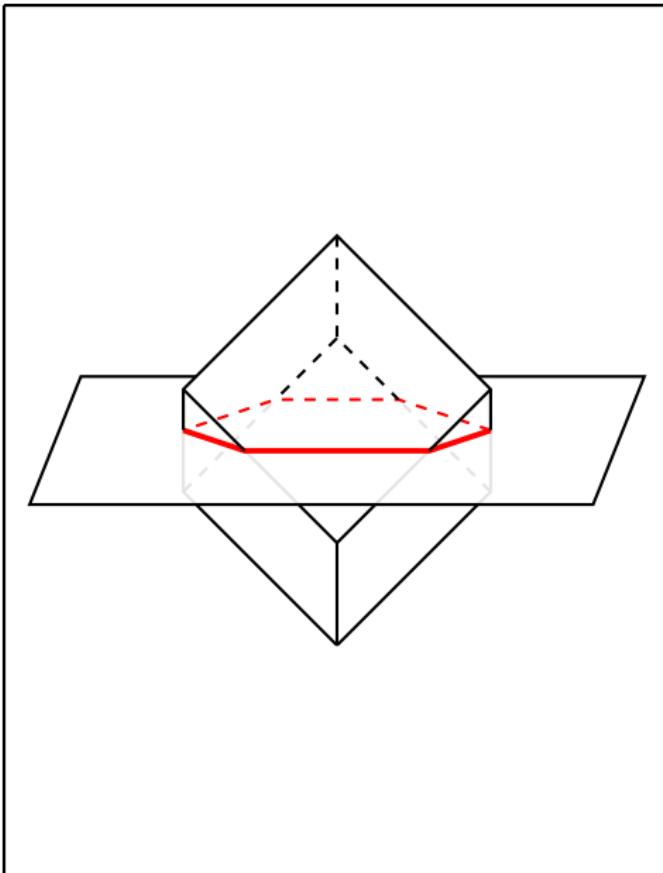
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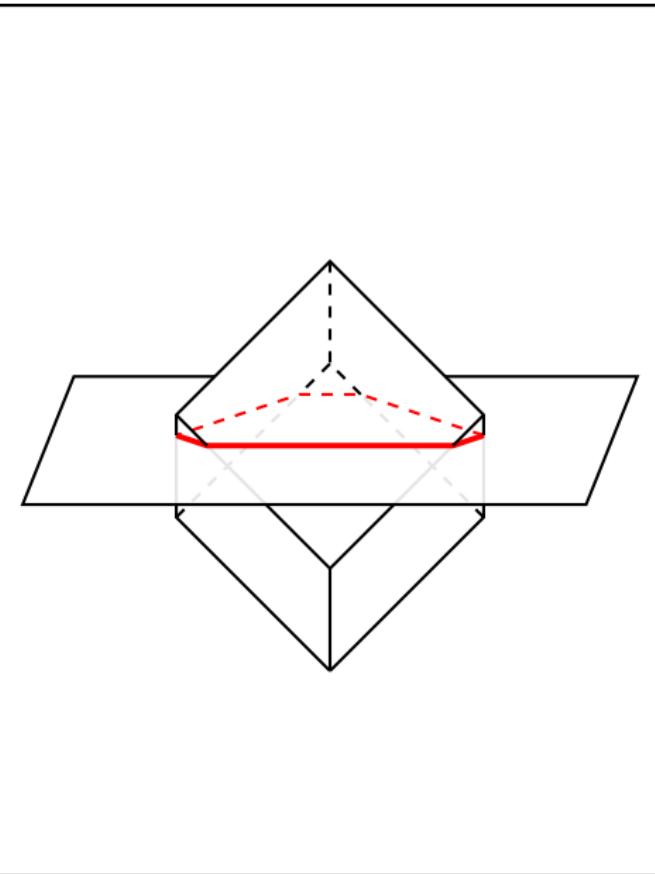
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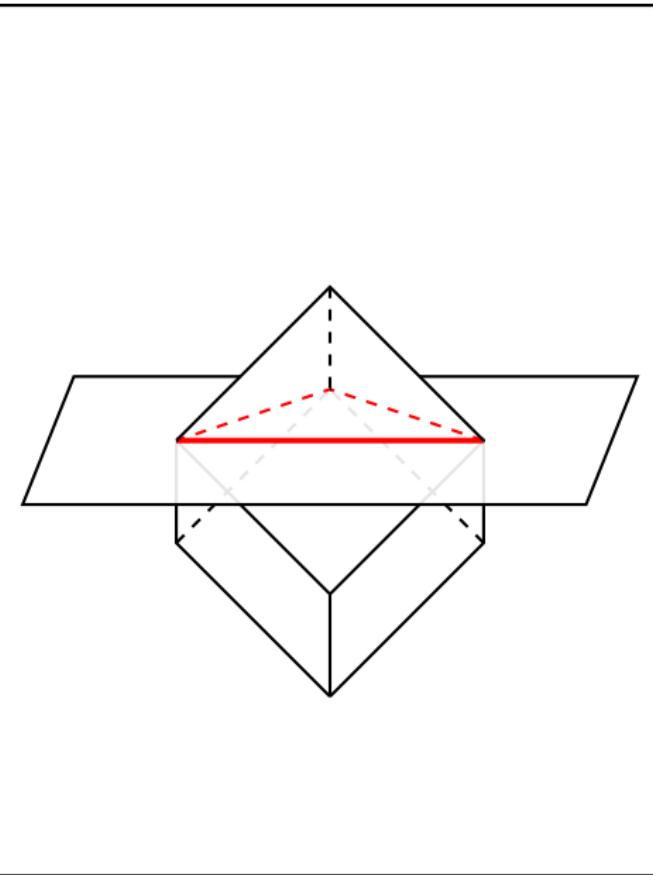
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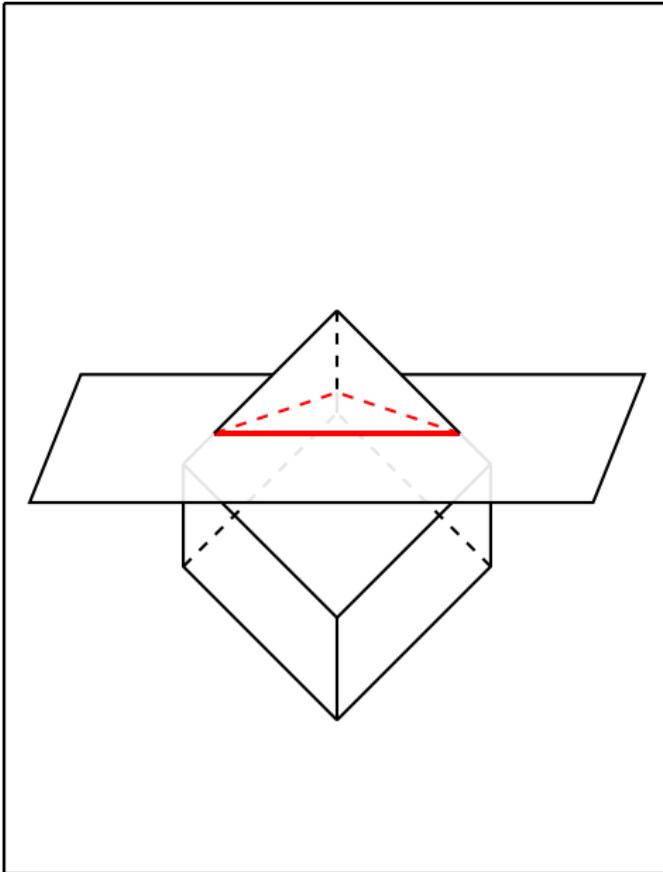
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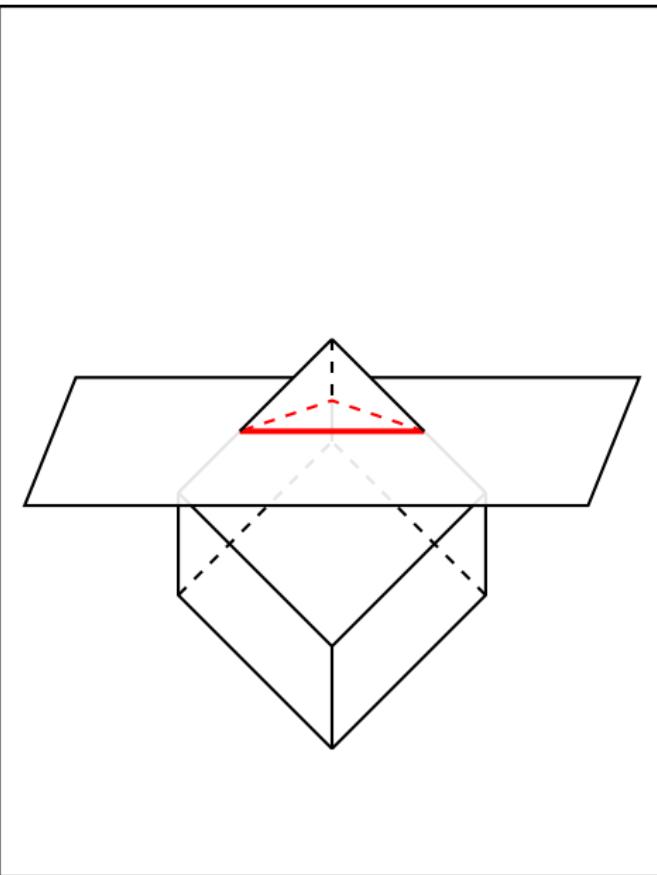
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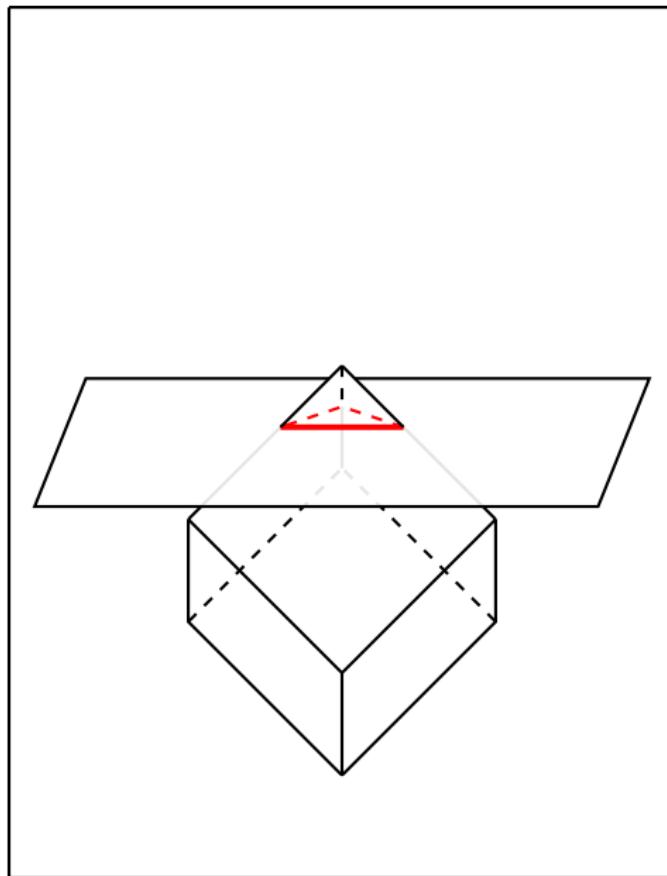
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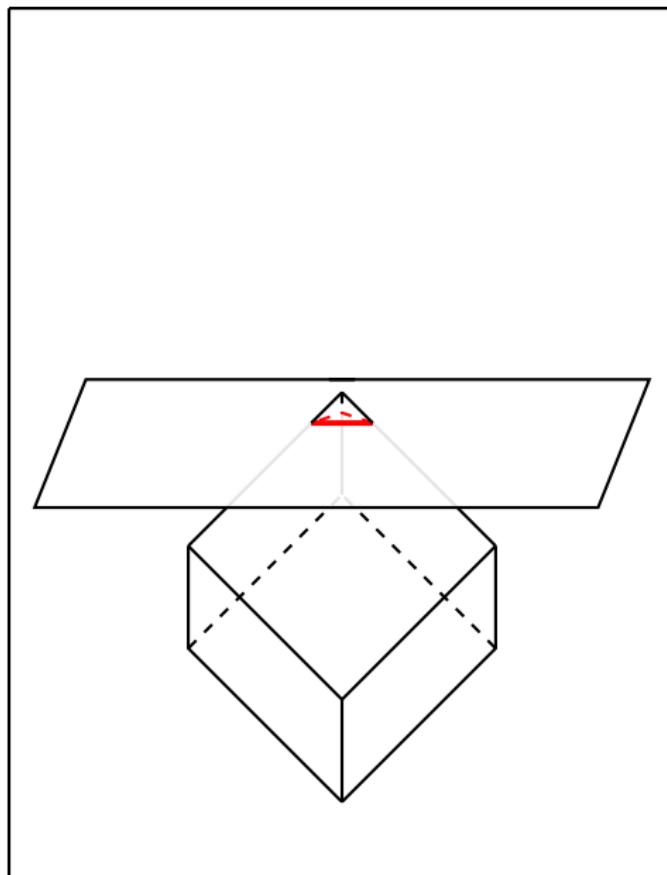
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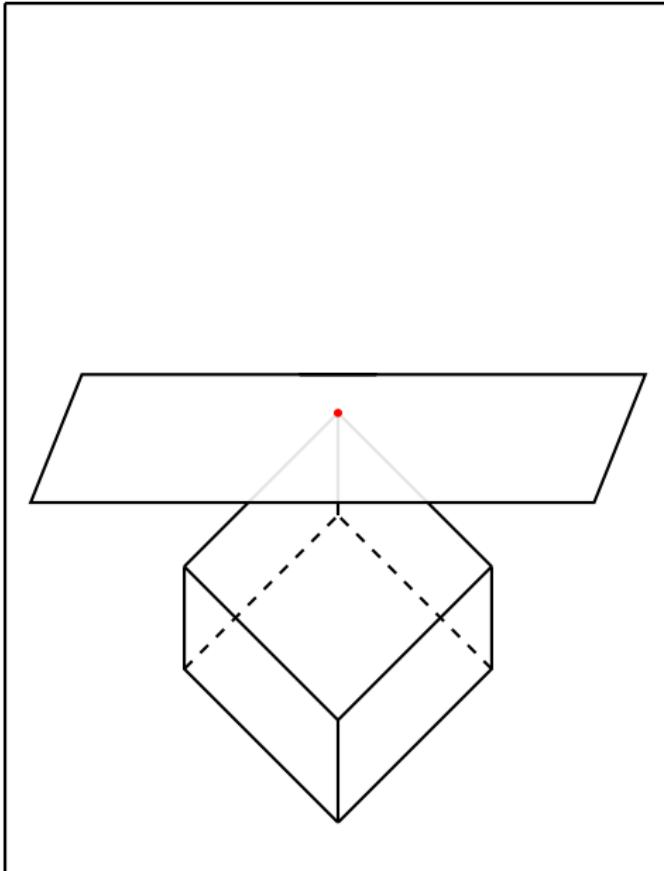
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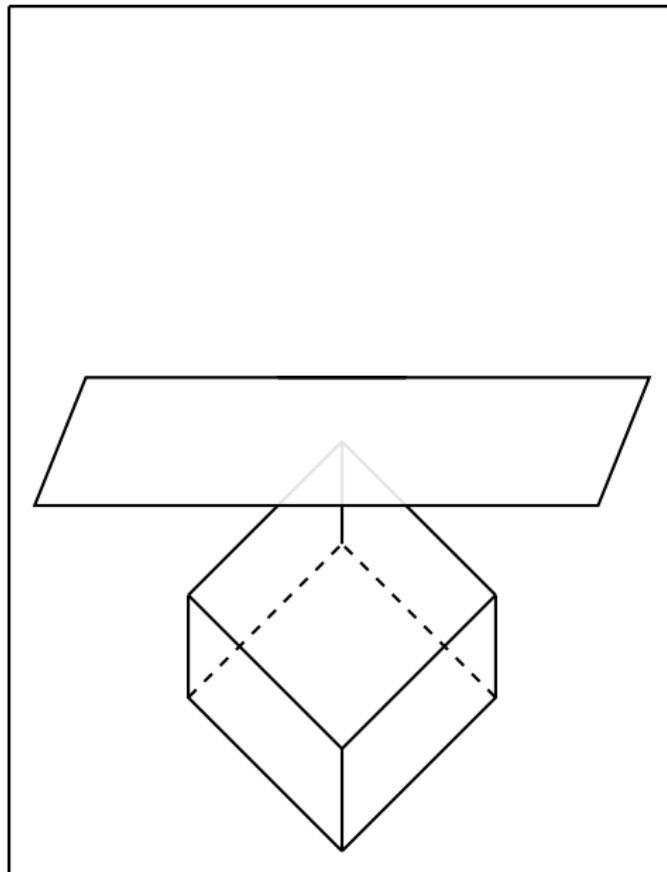
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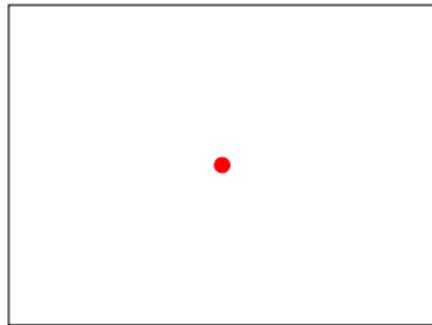


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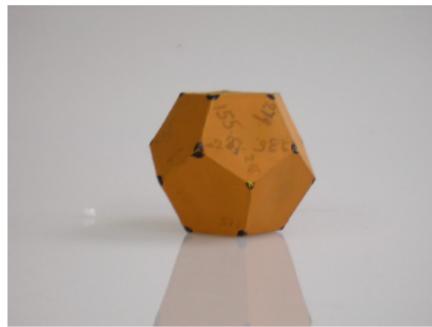




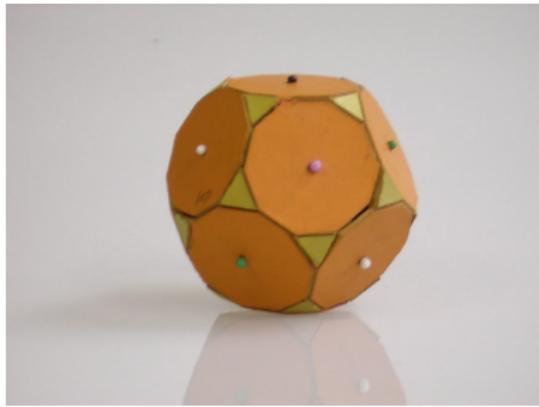
*Models of 3-D cross section of 4-D dodecahedron,*

Alicia Boole Stott, c. 1900

Collection of University of Groningen



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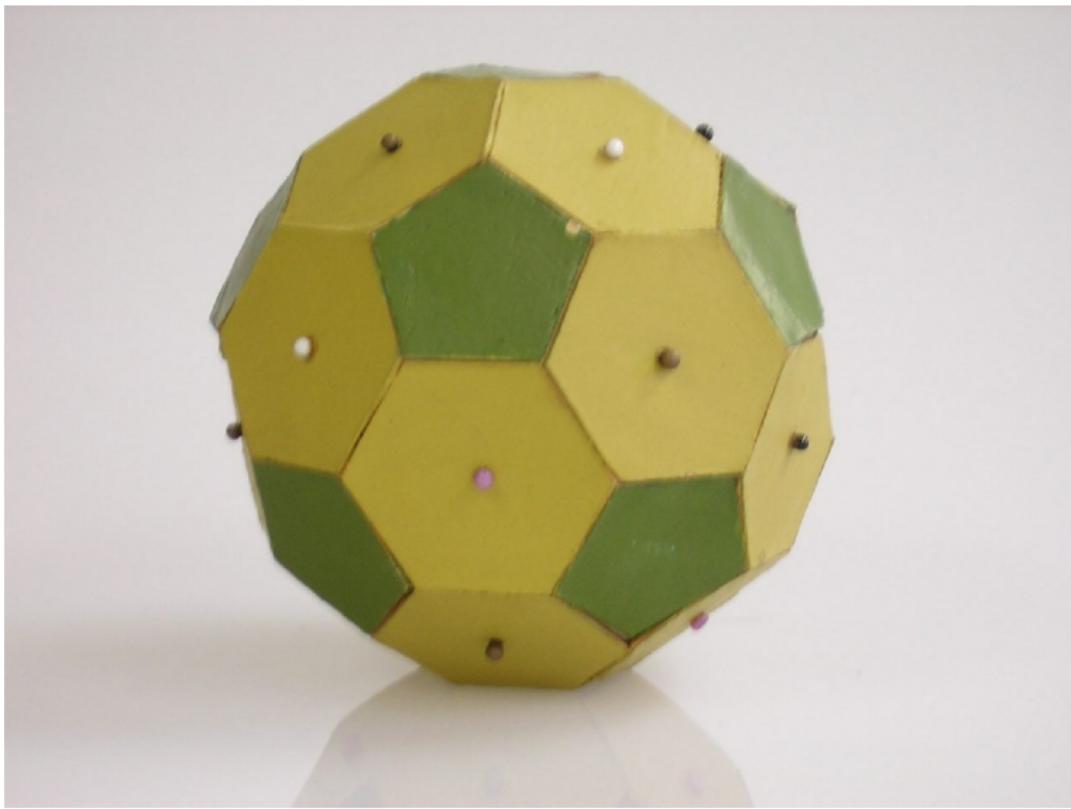
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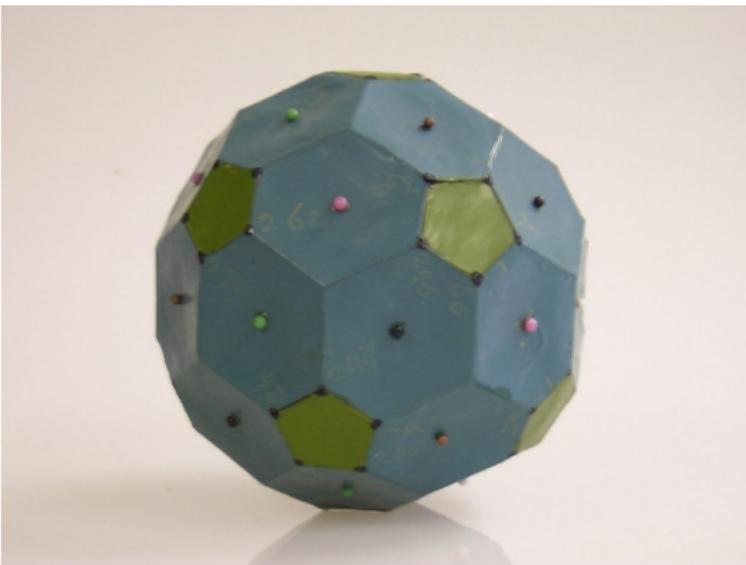
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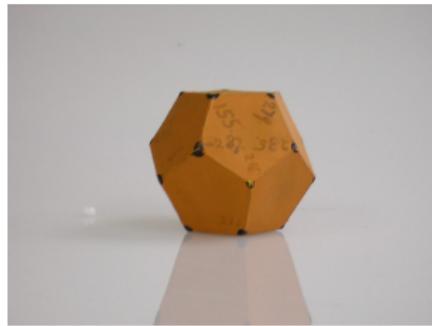
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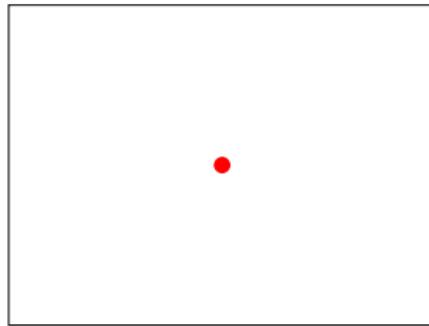
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## 4-D Polytope Slicer

See: [dogfeathers.com](http://dogfeathers.com)

<http://dogfeathers.com/java/hyperstar.html>

*Traité élémentaire géométrie à quatre dimensions,*  
by Esprit Jouffret, 1903

TRAITÉ ÉLÉMENTAIRE  
DE  
GÉOMÉTRIE A QUATRE DIMENSIONS

ET INTRODUCTION  
A LA GÉOMÉTRIE A  $n$  DIMENSIONS,

PAR

E. JOUFFRET,

Lieutenant-Colonel d'Artillerie en retraite,  
Ancien élève de l'École Polytechnique,  
Officier de la Légion d'honneur,  
Officier de l'Instruction publique,  
Membre de la Société mathématique de France.



PARIS,

GAUTHIER-VILLARS, IMPRIMEUR-LIBRAIRE  
DU BUREAU DES LONGITUDES, DE L'ÉCOLE POLYTECHNIQUE,  
Quai des Grands-Augustins, 35.

—  
1903

120

CHAPITRE VII.

donnent immédiatement, comme projection sur le plan des  $x_1 x_2$ , c'est-à-dire dans le compartiment A de la figure 23, le carré qui est dessiné avec des traits pleins et des points noirs, et dont les côtés sont parallèles aux axes. Les quatrième et cinquième colonnes donnent un carré pareil dans le compartiment C. Les

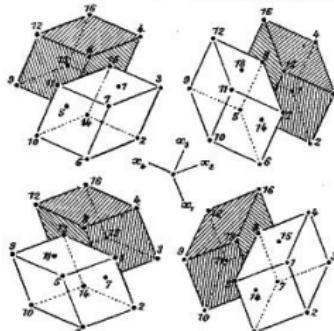


Fig. vi. — Les huit faces hexaédriques qui limitent l'octaèdre.

quatre axes de l'hypercube, que nous désignerons par  $a, b, c, d$ , sont parallèles, dans cette position, aux quatre axes coordonnés.

Que le lecteur veuille bien porter son attention sur le sommet inférieur droit du premier carré, qui porte les numéros 1, 2, 3, 4 et que nous reproduisons en a dans la figure 25; ce que nous en dirons s'appliquera à un quelconque des sept autres sommets. En ce point sont réunies les projections de quatre points de l'hypercube, qui ne peuvent être autre chose que les sommets d'une des vingt-quatre faces carrées, située dans un plan complètement

corps II; les deux premières, dont chacune a son plan perpendiculaire aux équateurs de huit octaèdres, se déduisent du Tableau I et fournissent les deux autres comme il est dit dans le Para-

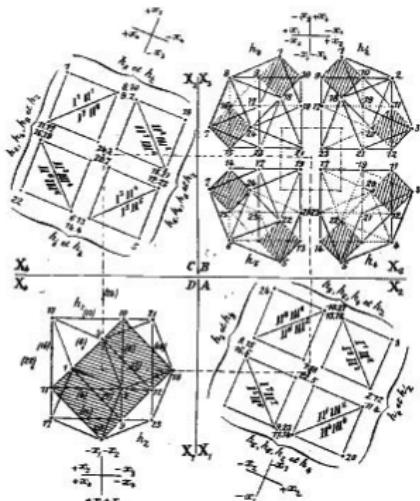
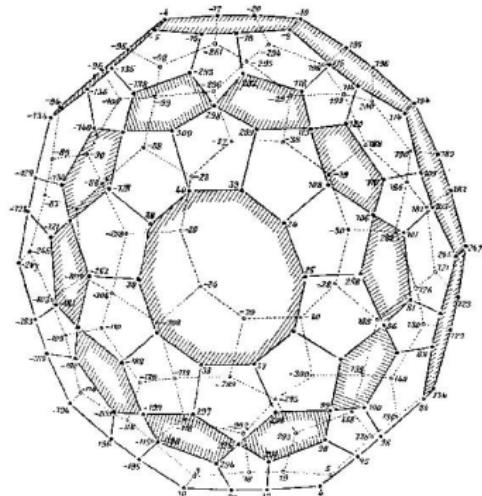


Fig. 40. — Projections planes des seize octaèdres fondamentaux.

graphie précédent. On voit, dans A et C, quatre carrés que nous avons un peu écartés les uns des autres une fois la construction faite; dans l'épure régulière, ils devraient se toucher, leurs quatre angles intérieurs coïncidant avec le point central de la figure. Les groupes de chiffres, qui sont, comme toujours, les



Maurice Princet (1875–1973) introduced Picasso to this book

Effect on Picasso \*

## Effect on Picasso \*



## Effect on Picasso \*



+

## Effect on Picasso \*



1

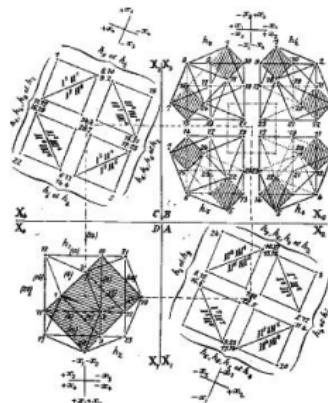


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# Effect on Picasso \*

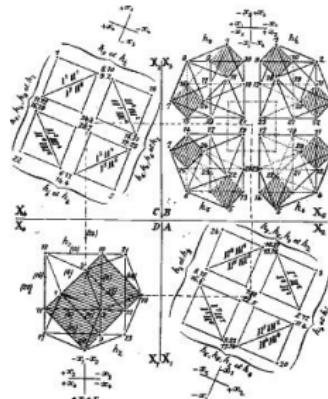


Fig. 5a. — Projections planes des neuf octaèdres fondamentaux.

## Effect on Picasso \*

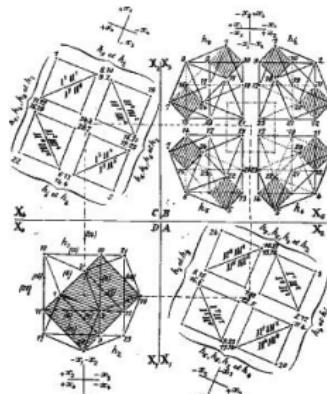


Fig. 5a. — Projections planes des neuf octaèdres fondamentaux.



## Effect on Picasso \*

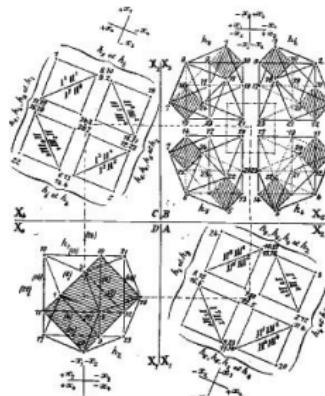


Fig. 50. — Projections planes des neuf octaèdres fondamentaux.



\* According to some art historians.

numéros des sommets correspondants de l'hypercorps, se rapportent également aux deux ou aux quatre points dont ils sont

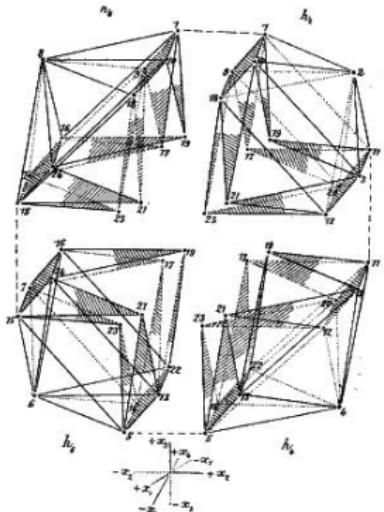


Fig. 6. — Perspective caudale des deux ostéoblastes fondamentaux.

voisins; nous ne les avons écrits qu'une fois, et il appartient au lecteur de faire, le cas échéant, le triage des numéros se rapportant à l'un ou à l'autre de ces points. Chaque carré de A ou C,

Pablo Picasso

*Portrait of Ambroise Vollard, 1910*



LES POLYÉDROÏDES.

153

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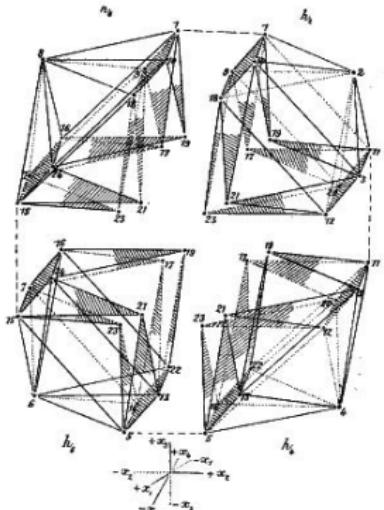


Fig. 45. — Perspective cavalier des sept octaèdres fondamentaux.

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## I. — Méthode géométrique.

La première et la deuxième colonnes du Tableau ci-dessus nous

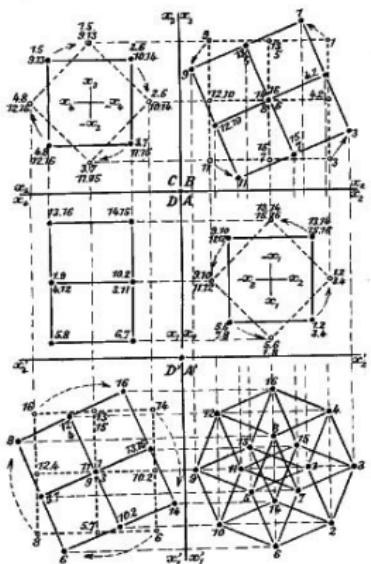


Fig. 12. — Diverses projections de l'octaédroïde, ou  $C'$ .

### I. — Mittwoch abonnierte.

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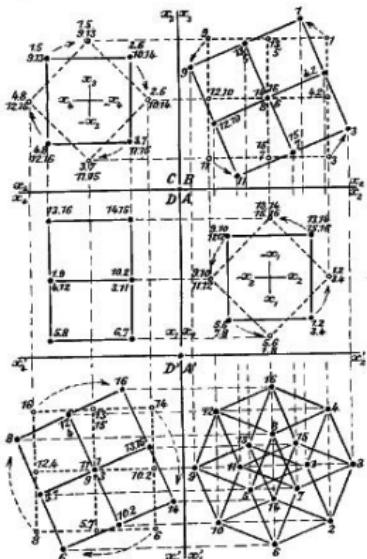


Fig. 32. — Diverses projections de l'octaédroïde, ou C<sub>8</sub>.

Pablo Picasso

*Portrait of Henry Kahnweiler, 1910*





Maurice de Vlaminck

*Restaurant de la Machine at Bougival*, 1905



*Still Life*, 1910



*Restaurant de la Machine at Bougival*, 1905

## Maurice de Vlaminck

"I witnessed the birth of cubism, its growth, its decline. Picasso was obstetrician, Apollinaire was the midwife, Princet was the godfather."



*Still Life*, 1910

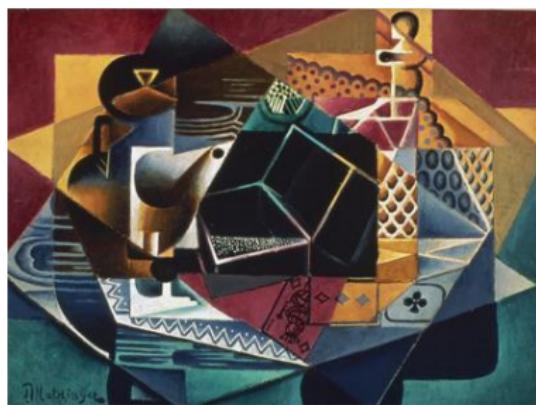


Jean Metzinger, *Cubist Landscape*, 1918



Jean Metzinger

Jean Metzinger, *Cubist Landscape*, 1918



*Cubist Still Life*, 1918



Jean Metzinger, *Cubist Landscape*, 1918



*Cubist Still Life*, 1918

## Jean Metzinger

"Picasso lays out a free, mobile perspective, from which that ingenious mathematician Princet has deduced a whole geometry"



Jean Metzinger, *Cubist Landscape*, 1918



*Cubist Still Life*, 1918

## Jean Metzinger

"Picasso lays out a free, mobile perspective, from which that ingenious mathematician Princet has deduced a whole geometry"

"Maurice Princet joined us quite often. Although quite young, thanks to his knowledge of mathematics he had an important job with an insurance company. But, beyond his profession, it was as an artist that he conceptualized mathematics, as an aesthetician that he invoked  $n$ -dimensional continuums. He loved to get artists interested in the new views on space that had been opened up by Schlegel and some others. He succeeded at that."



**Max Weber,**  
*Interior of the Fourth Dimension*, 1913



**Max Weber,**  
*Interior of the Fourth Dimension*, 1913

"The interior of the fourth dimension is the space around an art form which is stirred by the essence with which that form was vested by the artist."



**Max Weber,**  
*Rush Hour New York,*  
1915



**Max Weber,**  
*Rush Hour New York,*  
1915

"In plastic art, I believe, there is a fourth dimension which may be described as the consciousness of a great and overwhelming sense of a space-magnitude in all directions at one time, and is brought into existence through the three known measurements. It is not a physical entity or a mathematical hypothesis, nor an optical illusion. It is real and can be perceived and felt. It exists outside and in the presence of objects, and is in the space that envelops a tree, a tower, a mountain or any solid; or in the intervals between objects or volumes of matter if receptively beheld. It is somewhat similar to color and depth in musical sounds. It arouses imagination and stirs emotion. It is the immensity of all things. It is the ideal measurement, and is therefore as great as the ideal, perceptive or imaginative faculties of the creator, architect, sculptor or painter.



Max Weber, *New York at Night*, 1915



**Albert Gleizes**

*Landscape with a Figure*, 1911



**Albert Gleizes**  
*Landscape with a Figure*, 1911



**Albert Gleizes**  
*Women in a Kitchen*, 1911

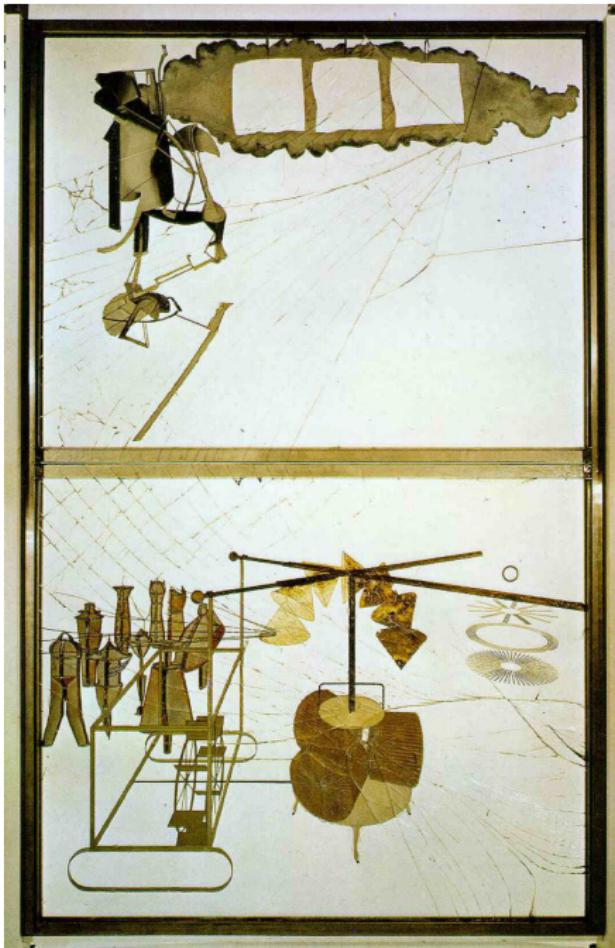


**Albert Gleizes,**  
*Brooklyn Bridge*, 1915



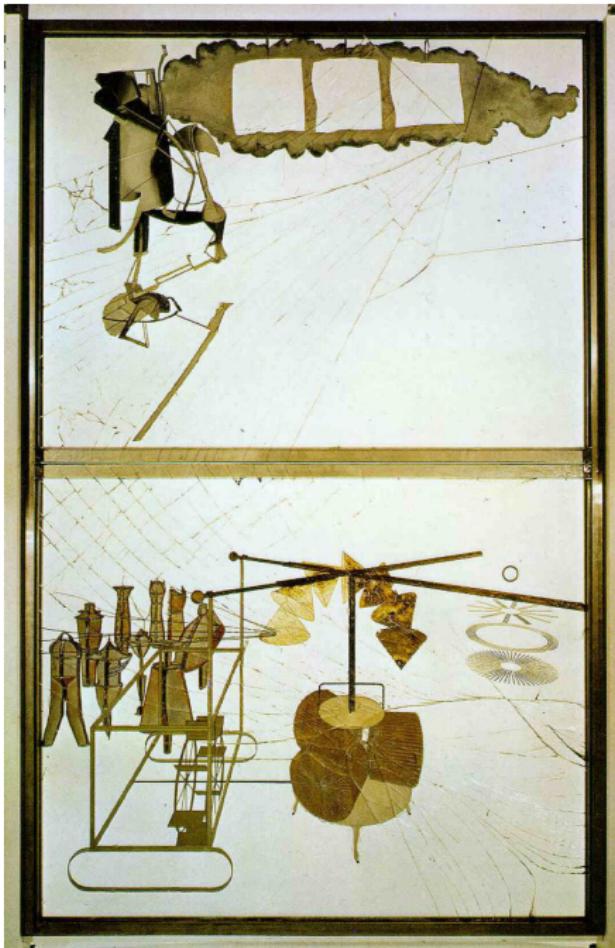
**Albert Gleizes,**  
*Brooklyn Bridge*, 1915

"But beyond the three dimensions of Euclid, we have added another, the *fourth dimension* which is to say the figuration of space, the measure of the infinite."



## Marcel Duchamp,

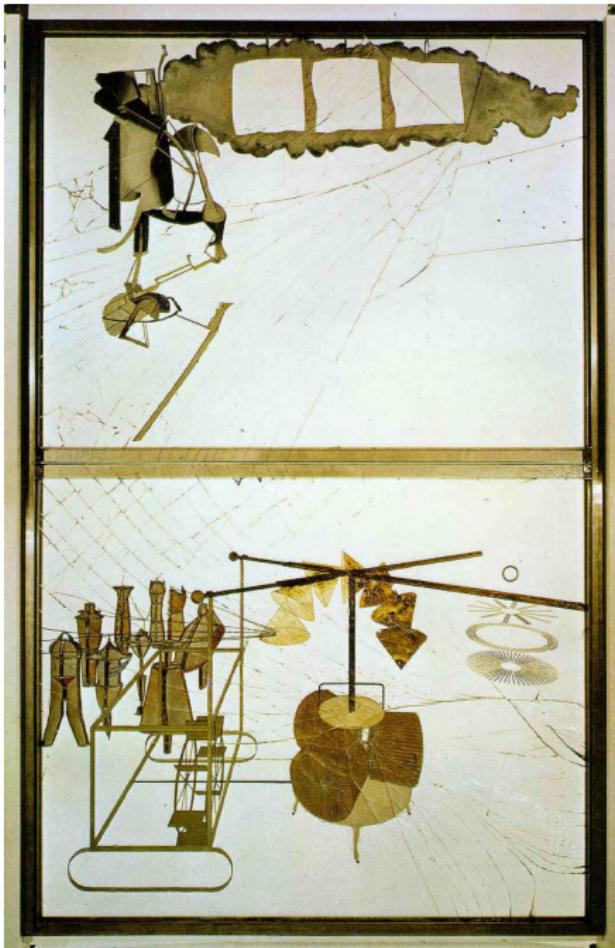
*The Bride Stripped Bare by Her Bachelors, Even  
(The Large Glass)*,  
1915–1923



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*The Bride Stripped Bare by Her Bachelors, Even  
(The Large Glass)*,  
1915–1923

The ideas in the Large Glass are more important than the actual realization.

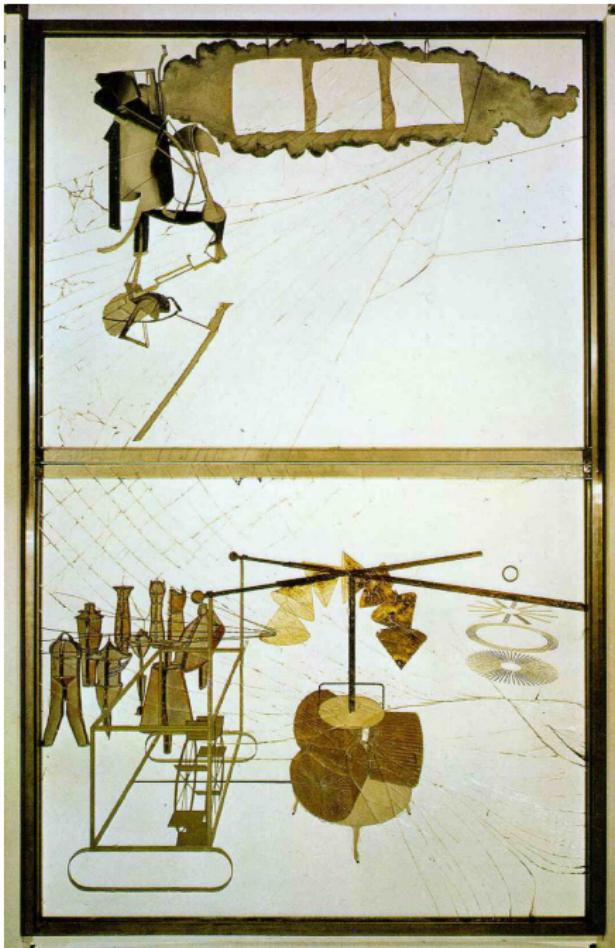


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1915–1923

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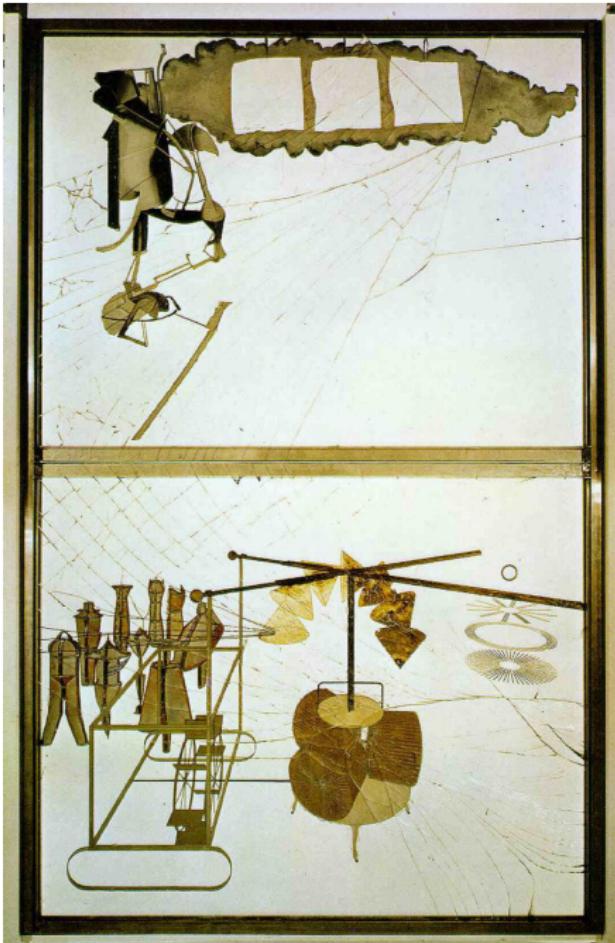
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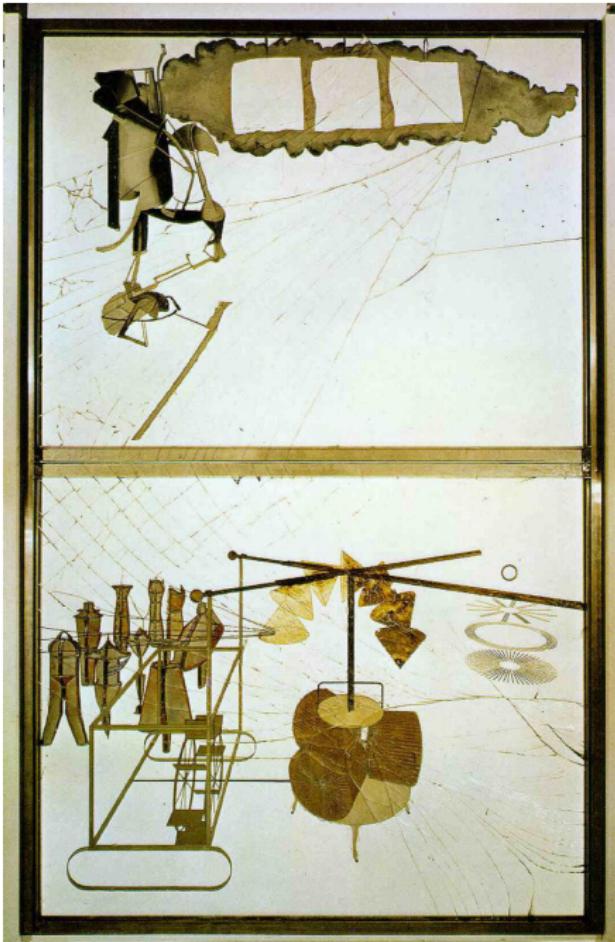
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Since I found that one could cast a shadow from a three-dimensional thing, any object whatsoever – just as the projecting of the sun on the Earth makes two dimensions – I thought that, by simple intellectual analogy, ...any three-dimensional object...is a projection of something four-dimensional, something we're not familiar with. "The Bride" in the "Large Glass" was based on this, as if it were the projection of a four-dimensional object.



## Marcel Duchamp,

*The Bride Stripped Bare by Her Bachelors, Even  
(The Large Glass)*,  
1915–1923

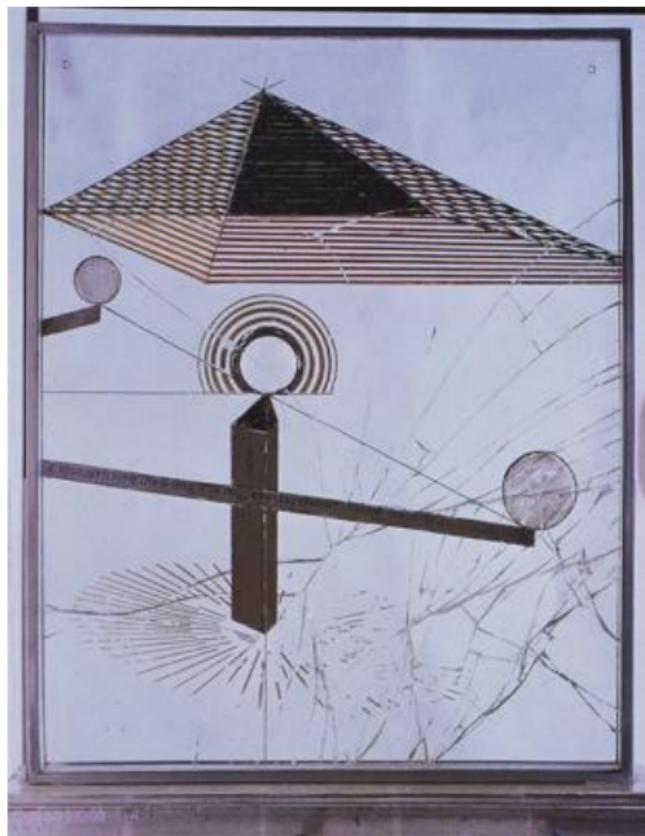
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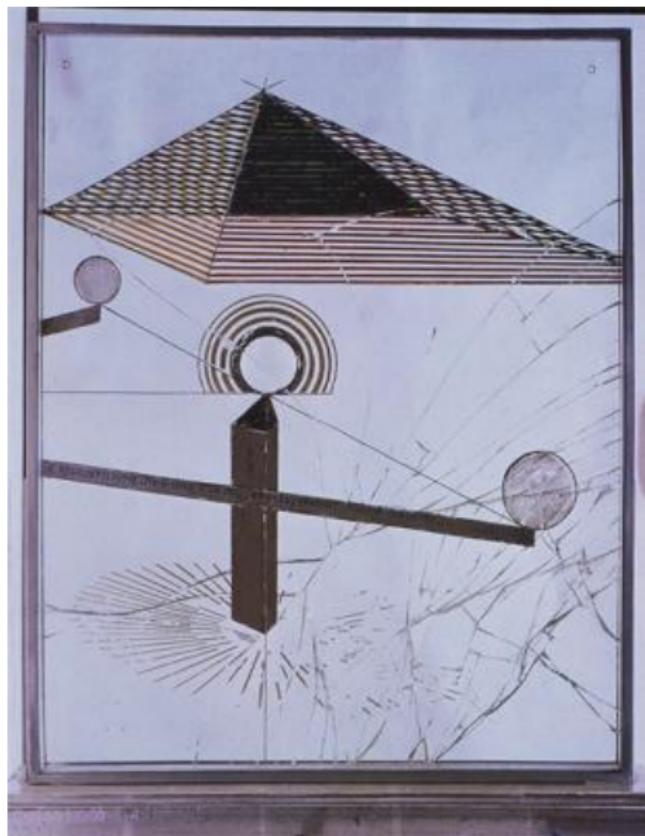
What we were interested in at the time was the fourth dimension. Simply, I thought of the idea of a projection, of an invisible fourth dimension, something you couldn't see with your eyes.

Since I found that one could cast a shadow from a three-dimensional thing, any object whatsoever – just as the projecting of the sun on the Earth makes two dimensions – I thought that, by simple intellectual analogy, ...any three-dimensional object...is a projection of something four-dimensional, something we're not familiar with. "The Bride" in the "Large Glass" was based on this, as if it were the projection of a four-dimensional object.

See <http://www.understandingduchamp.com/>



**Marcel Duchamp,**  
*To Be Looked At (from the Other Side of  
the Glass, with One Eye Close to, for Al-  
most an Hour...)*,  
1918



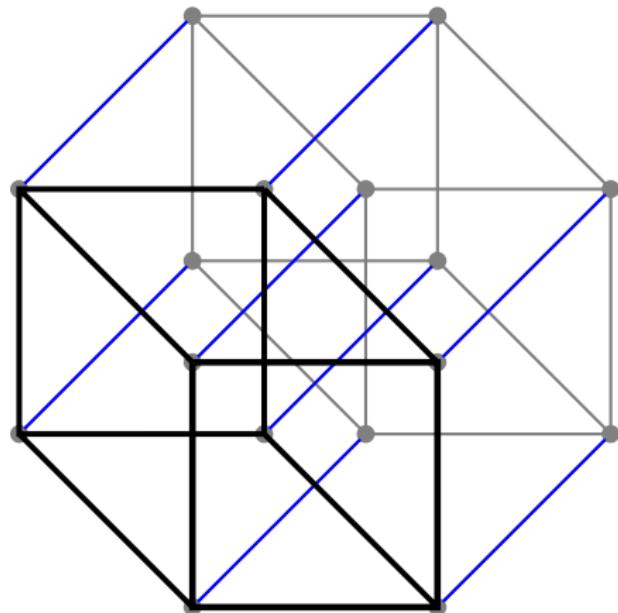
**Marcel Duchamp,**  
*To Be Looked At (from the Other Side of  
the Glass, with One Eye Close to, for Al-  
most an Hour...)*,  
1918

"We weren't mathematicians at all, but we really did believe in Princeton."

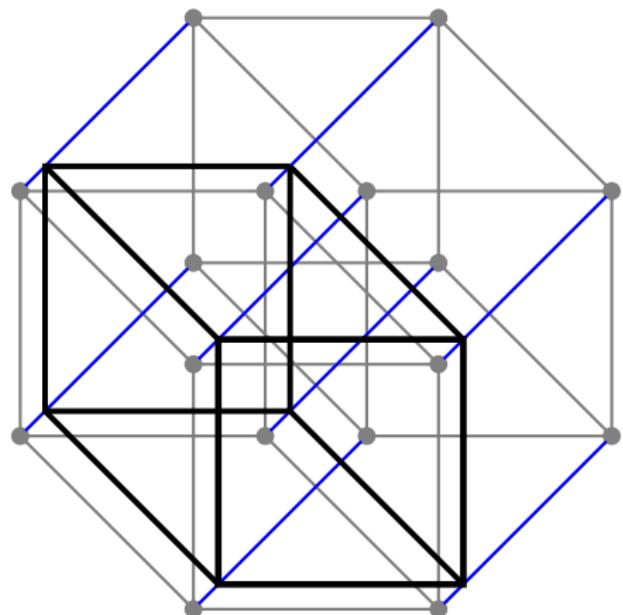


**Marcel Duchamp,**  
*Nude Descending a Staircase No. 2*,  
1912

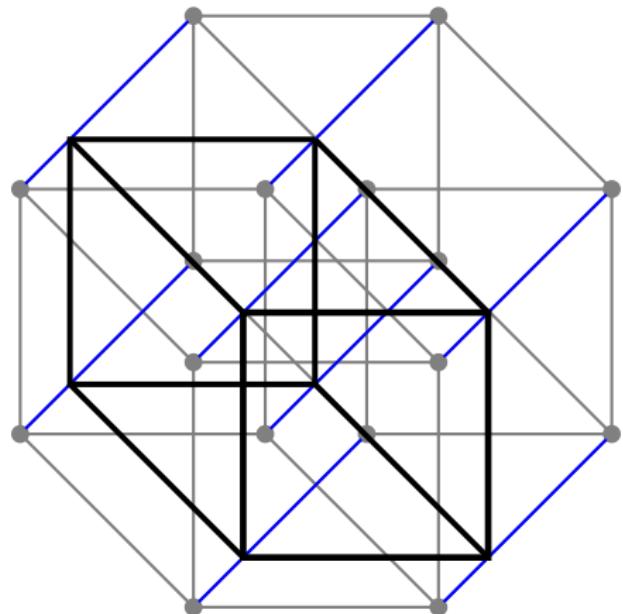
## The Fourth Dimension as Time



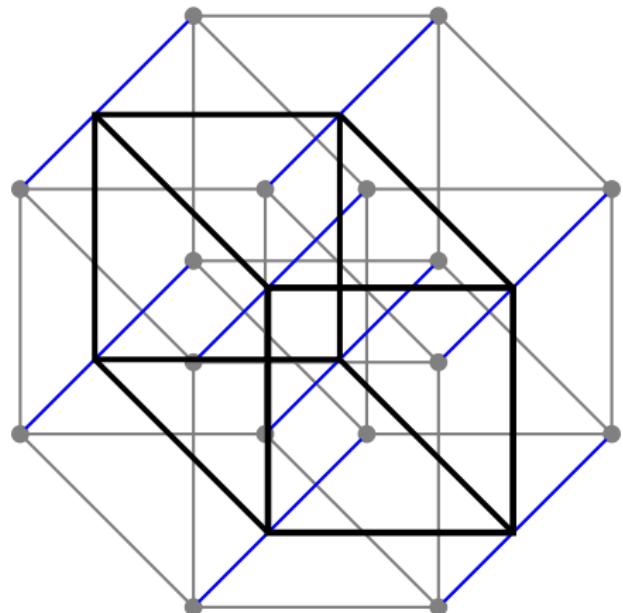
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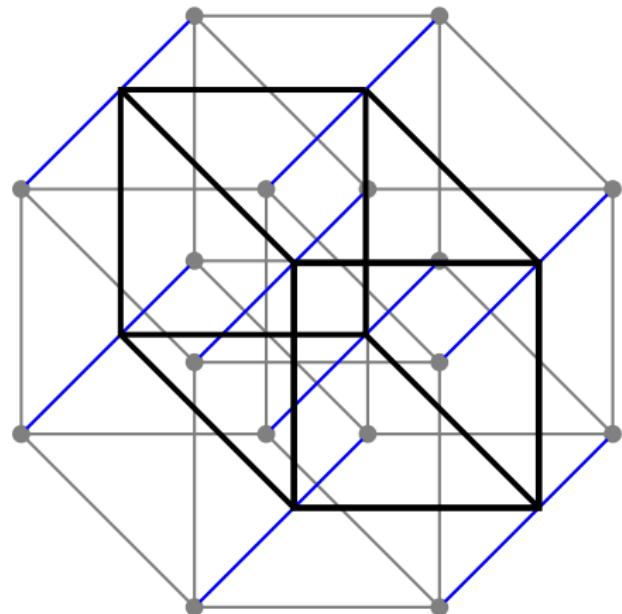
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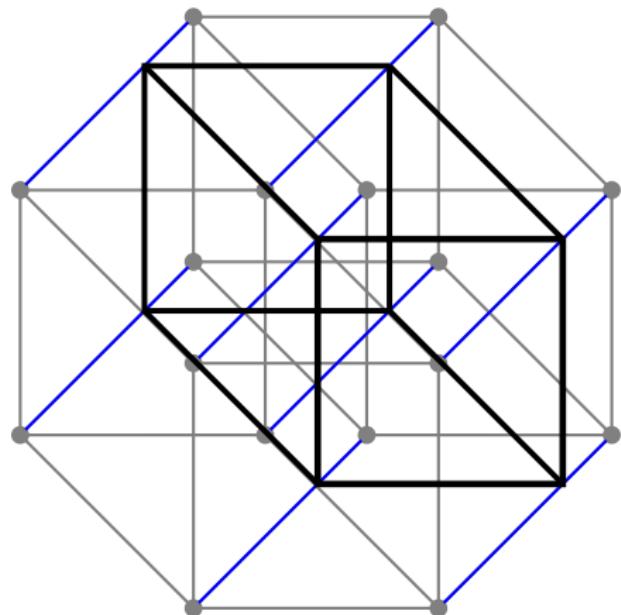
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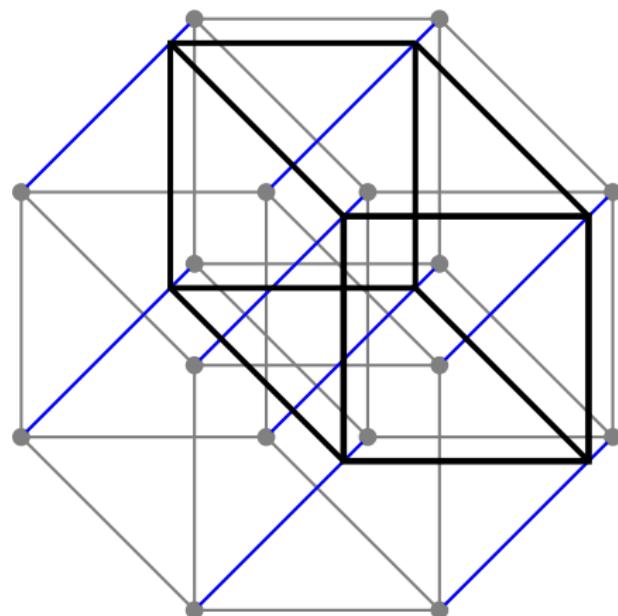
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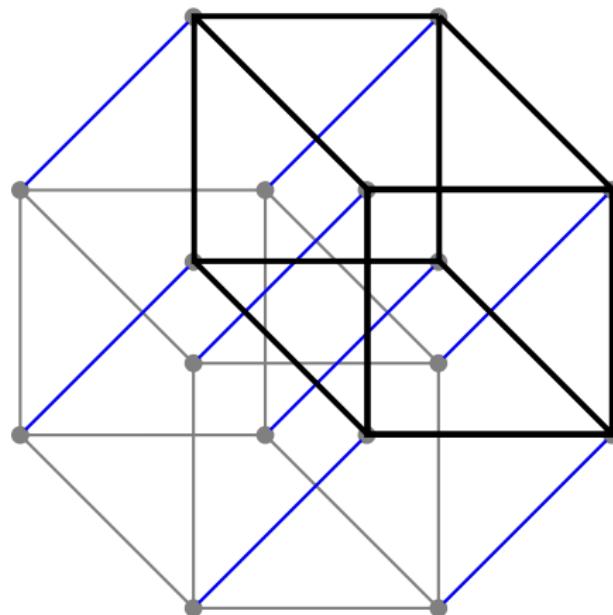
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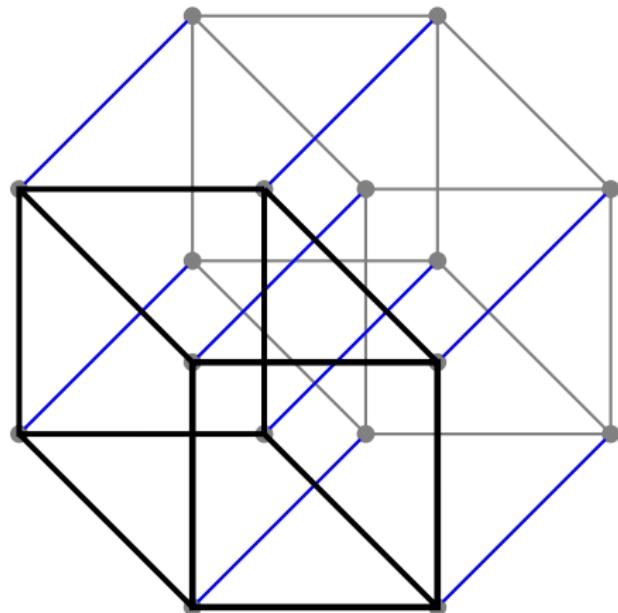
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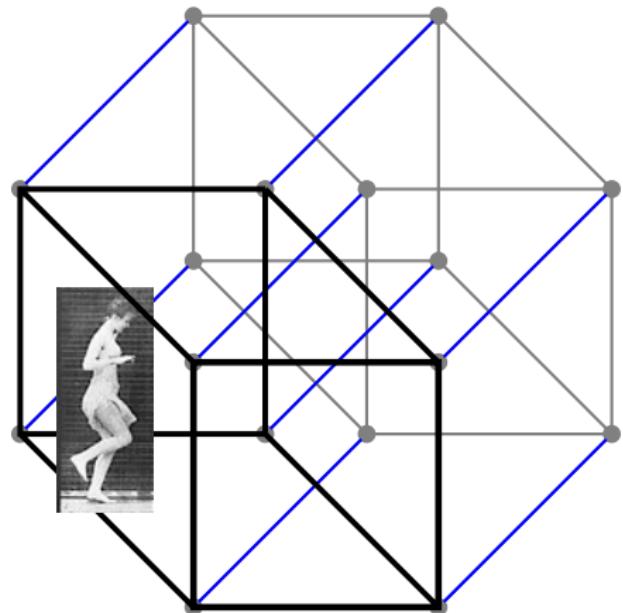
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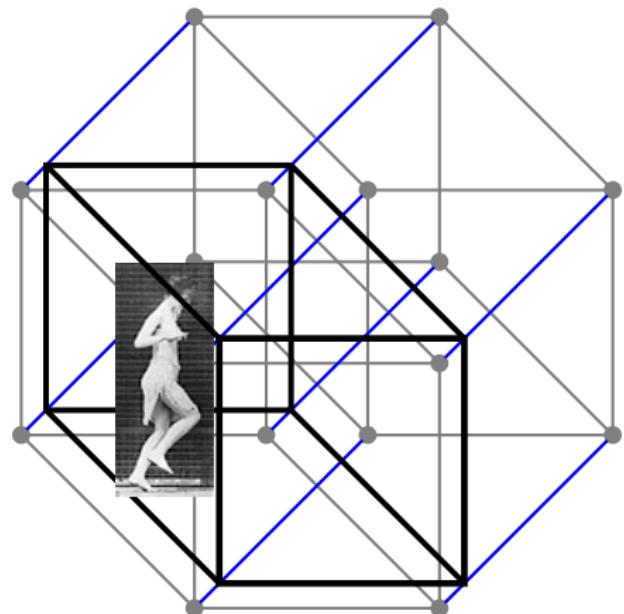
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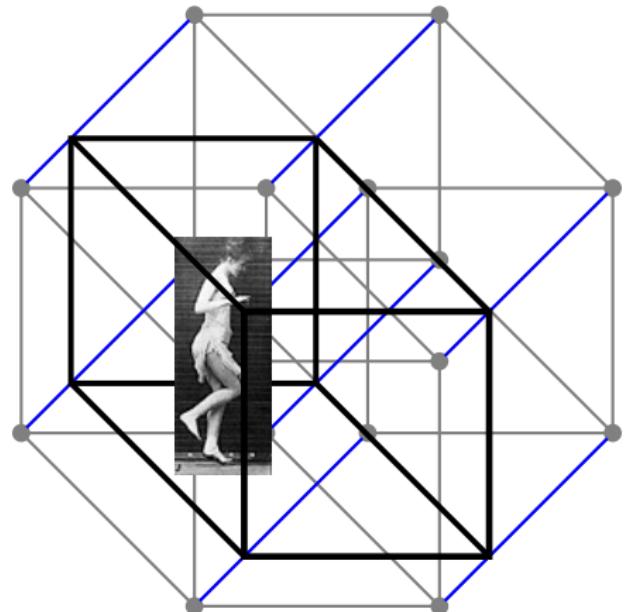
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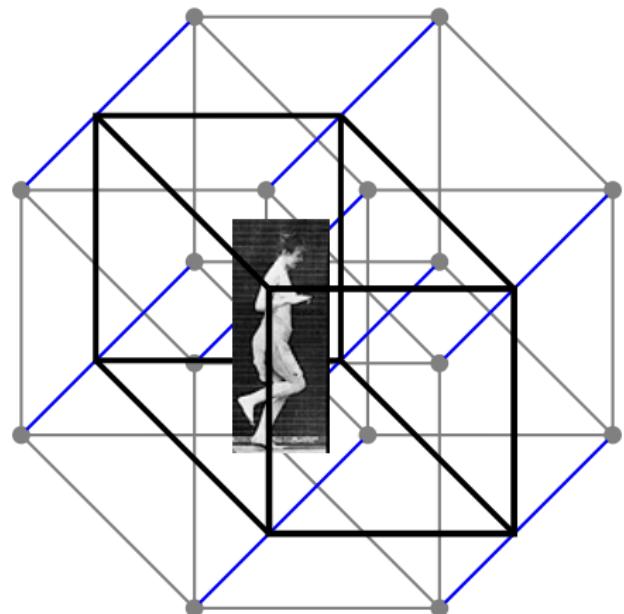
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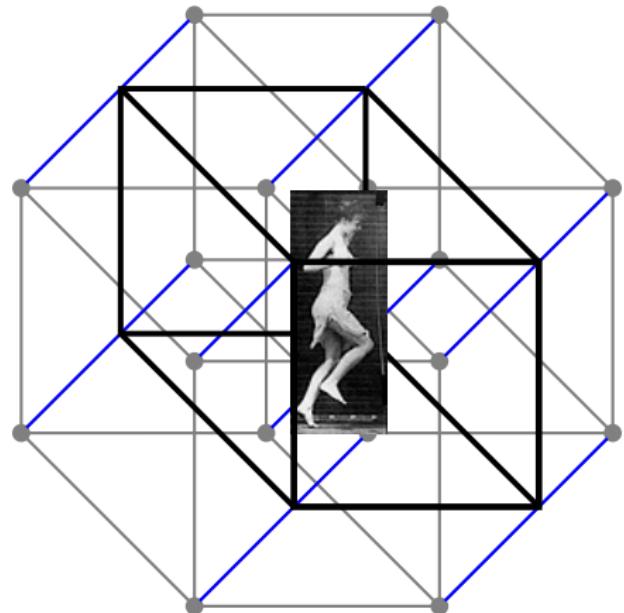
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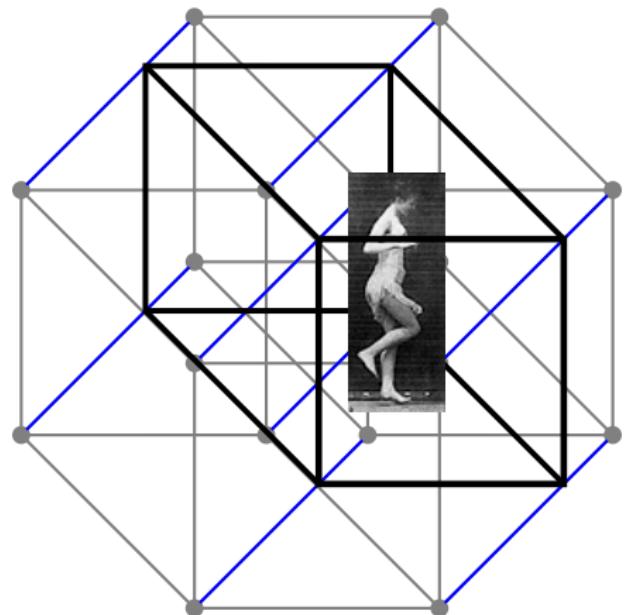
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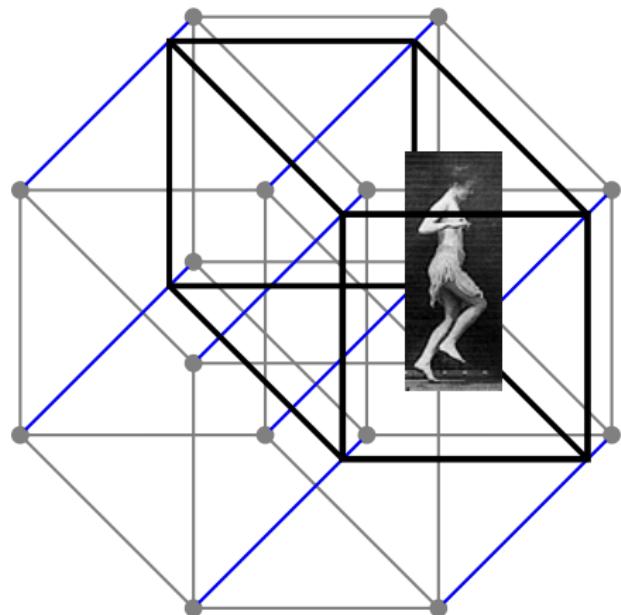
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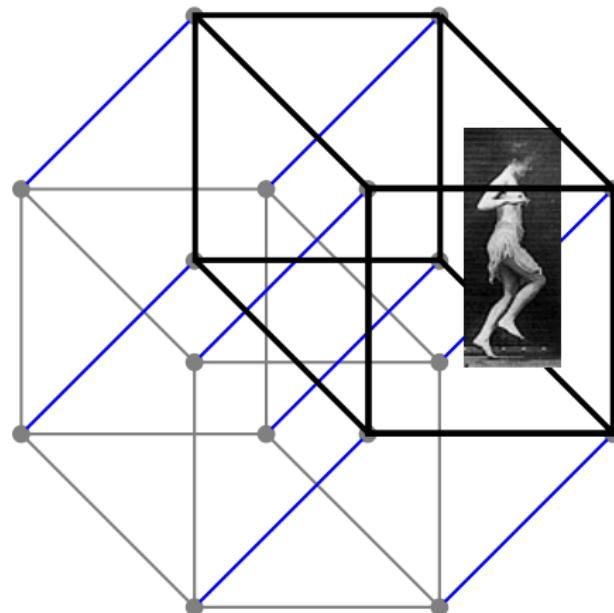
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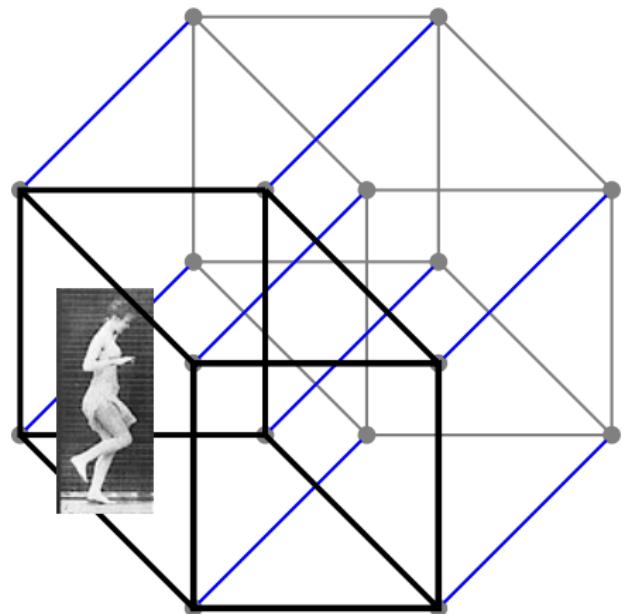
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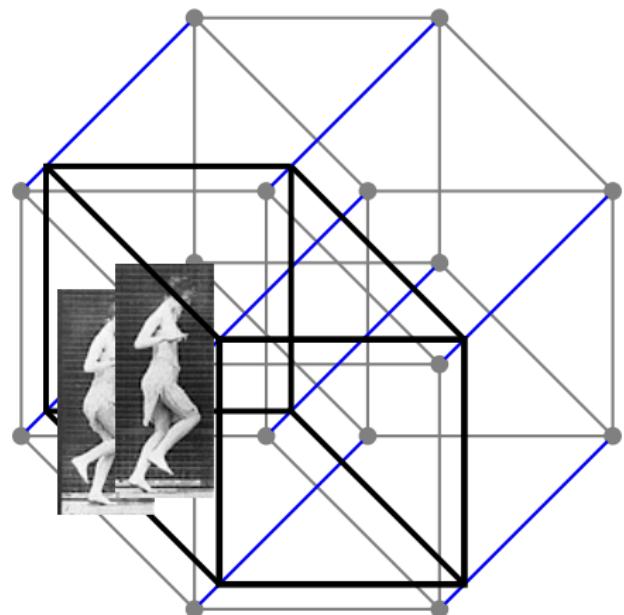
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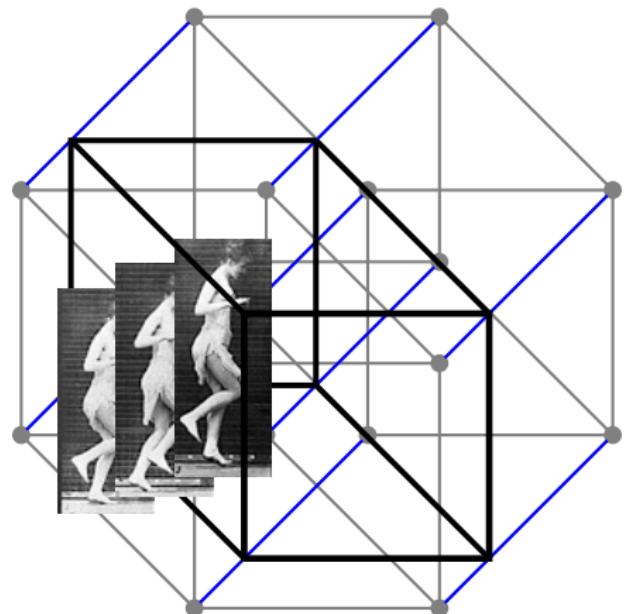
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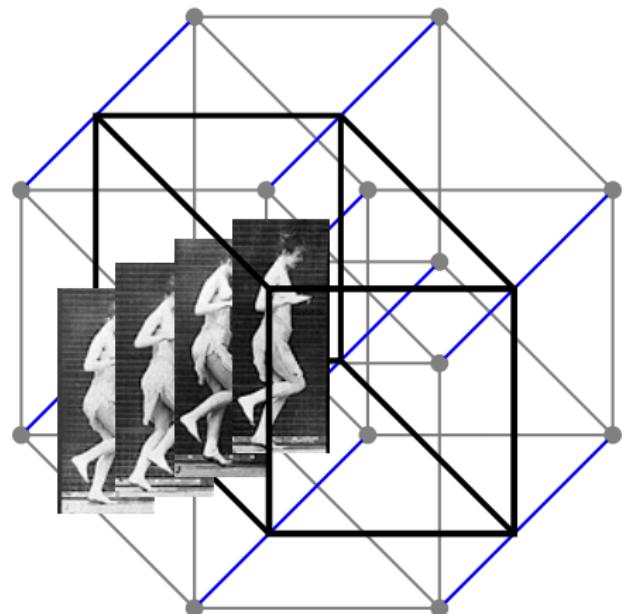
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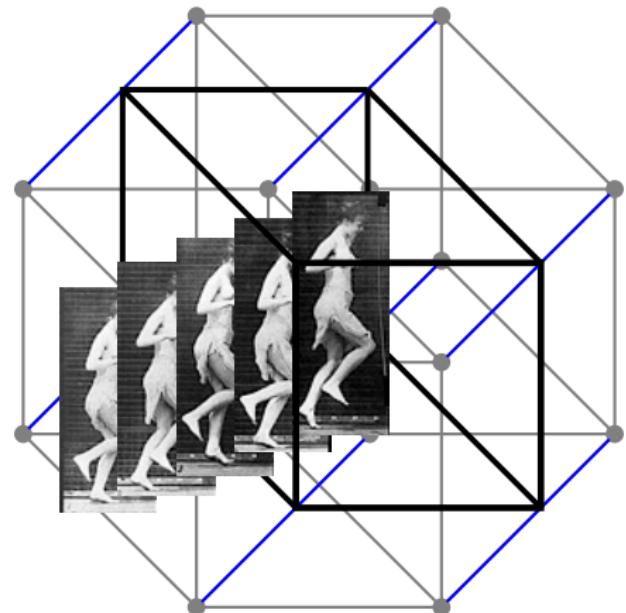
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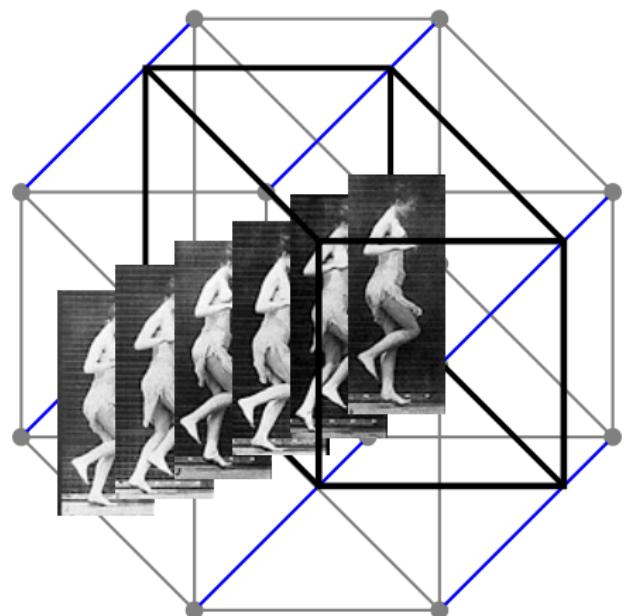
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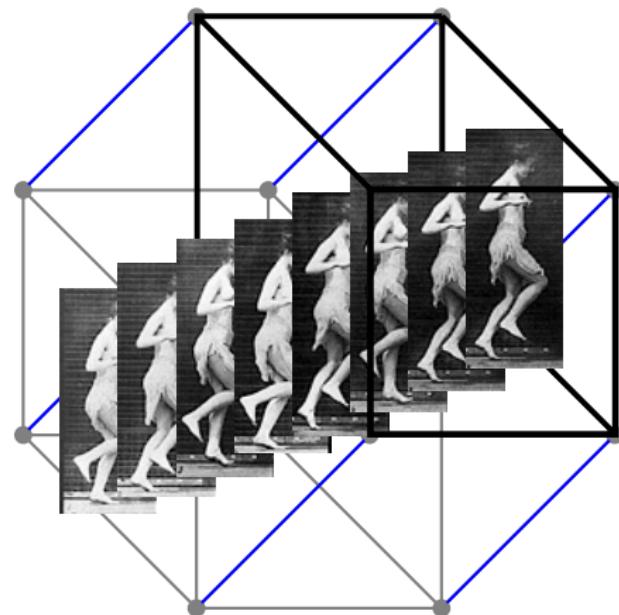
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## The Fourth Dimension as Time



## The Fourth Dimension as Time



Giacomo Balla

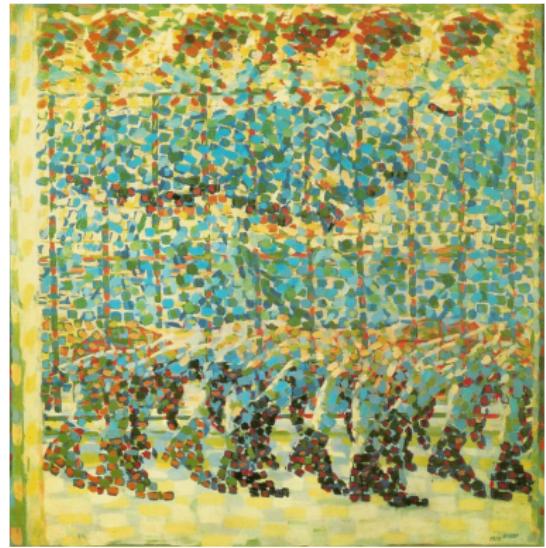


*Dynamism of a Dog on a Leash*  
oil on canvas, 1912

## Giacomo Balla



*Dynamism of a Dog on a Leash*  
oil on canvas, 1912



*Young Girl Running on a Balcony*  
oil on canvas, 1912

Next Time:

Non-Euclidean Geometry and the Visual Arts

<http://www.people.vcu.edu/~rhammack/Math123/>