**Last name** ______________________

**First name** ______________________

**LARSON—MATH 656—Test 2 Review**

Write up careful and complete answers.

**Concepts & Notation**. Give a careful definition and **example** for each concept.

1. What is a 1-factor? (And what is the difference from a perfect matching?)

2. Given a set $S \subseteq V(G)$, what is $o(G - S)$?

3. What is Tutte's Condition?

4. Given a set $S \subseteq V(G)$, what is $def(S)$?

5. What is $def(G)$?

6. What is a *factor-critical graph*?

7. What is a *near-perfect* matching?

8. What is a *feasible flow* (Give a non-0-flow example)?

9. What is the *value* ($val(f)$) of a flow $f$?

10. What is a *maximum flow*?

11. What is a $f$-*augmenting* path?

12. What is the *tolerance* of an $f$-augmenting path $P$?

13. What is a source/sink cut $[S, T]$ in a network?

14. What is the *capacity* ($cap(S, T)$) of a source/sink cut $[S, T]$ in a network?

**Theorems**

15. What is *Tutte's Theorem*?

16. What is the *Berge-Tutte Formula*?

17. What is *Petersen's Theorem*?

18. What is the *Max-Flow Min-Cut Theorem*?

**Proofs**.

19. Prove: Any matching $M$ in a graph $G$ with order $n$ leaves at least $def(G)$ vertices unsaturated.

20. Explain the *ideas* of the proof of the **Berge-Tutte Formula**.

21. Prove the **Weak Duality** Theorem: If $f$ is a feasible flow and [S,T] is a source/sink cut, then $val(f) \leq cap(S,T)$.

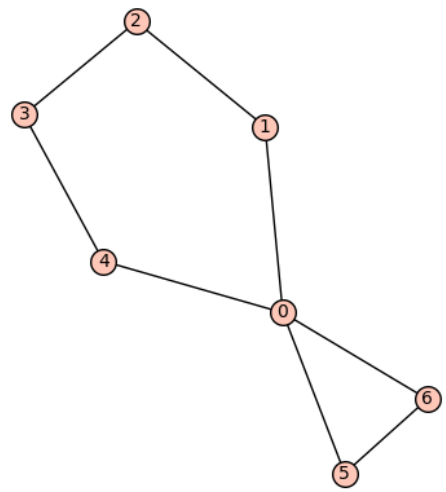
22. Prove: the *Max-Flow Min-Cut Theorem*

**Algorithms**

23. What is the main idea of Edmond's Blossom Algorithm?

24. What is the the *Ford-Fulkerson Algorithm* (Explain the steps)?

**Problems**. Explain as completely as you can.




Call this graph the *lantern*.

25. Find the sets $D$, $A$, and $C$ from the Gallai-Edmonds Decomposition for the lantern graph.

26. Find a maximum deficiency set $T$ for the lantern graph. Is it maximal?

27. Find the auxiliary graph $H(T)$ for the lantern graph and your set $T$.

28. Show that $def(A) = def(G)$ for the lantern graph.

29. Find the sets $D$, $A$, and $C$ in the complete bipartite graph $K_{m,n}$ from the Gallai-Edmonds Decomposition.

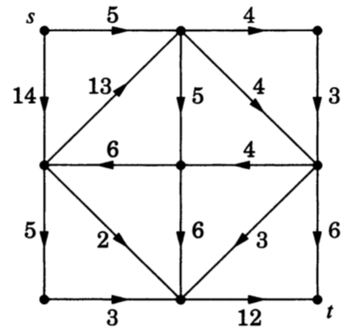
30. Find a maximum deficiency set $T$ in $K_{m,n}$.

31. Show that $def(A) = def(G)$ for $K_{m,n}$.

32. Find the sets $D$, $A$, and $C$ in the cycle graph $C_n$ (with $n$ even) from the Gallai-Edmonds Decomposition.

33. Find the sets $D$, $A$, and $C$ in the cycle graph $C_n$ (with $n$ odd) from the Gallai-Edmonds Decomposition.

34. Give an example that shows that Petersen's Theorem may not hold for a cubic graph with cut edges.




35. Find a maximum flow $f$ in this network (with indicated capacities) and argue that it is in fact maximum.

36. Find a minimum cut $[S, T]$ in this network.

37. How is the Max-Flow Min-Cut Theorem a Min-Max Relation and how does it provide a "certificate" for a maximum flow or a minimum cut?

38. How can network flow theory be used to find a maximum matching in a bipartite graph?