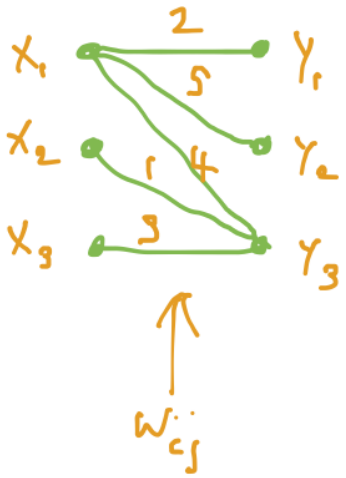
Last name _____

First name _____

LARSON—MATH 656—HOMEWORK 04
Hungarian Method Example.



Consider the (X, Y)-bigraph G above with edge weights $w_{i,j}$ indicated.
Use the Hungarian Method to find a maximum weight matching M and minimum cost cover (u, v).
Include all your steps.

1. View the given bigraph as $K_{3,3}$ with all missing edges having 0-weight.
2. Initialize a cover (u, v) where $u_i + v_j \geq w_{ij}$ for each edge.
   (a) Find the *equality subgraph* G_e.
   (b) Find a maximum cardinality matching M and minimum vertex cover Q in G_e (so $|M| = |Q|$). (Kuhn calls this the "Kőnig step").
   (c) If M is a perfect matching, then M is a maximum weight matching and (u, v) is a minimum weight cover.
   (d) Find R, T, $X - R$, and $Y - T$.
   (e) Find the minimum *excess* ϵ of all edges from $X - R$ to $Y - T$.
   (f) Update the cover: subtract ϵ from each u_i corresponding to $x_i \in X - R$, and add ϵ to each v_j corresponding to $y_j \in T$. (Kuhn calls this the "Egerváry step").
   (g) Repeat this loop (unil a perfect matching M is found in the Kőnig step.)
3. Prove that your matching M is maximum weight and cover (u, v) is minimum cost.