Last name ______________________

First name ______________________

LARSON—MATH 656—CLASSROOM WORKSHEET 26
Network Flows.

Organizational Notes

1. Don't forget to send your Notes / Classroom worksheet after each class (make the email subject useful: like "Math 656 c26 notes").
2. The VCU Discrete Math Seminar is every Wednesday.
3. Homework #7 is due today. Don't forget to send that.
4. Homework #8 is the Test 2 Review. That's due at 11:59 pm Tuesday night (May 4).
5. Test 2 is next Wednesday (May 5).
6. Read ahead! We're talking about Network Flow problems (Sec. 4.3)

Almost all graphs have Diameter 2

1. What is the ratio of the number of graphs with order n and diameter=2 to the number of all graphs with order n?
2. What is the ratio of the number of graphs with order n and which are complete to the number of all graphs with order n?
3. One way to think about these ratios is as probabilities: what is the probability of pulling a complete graph of order n from a hat with all graphs of order n? And one way to think about the probability is to think of a graph-generating process: flip a coin for each pair of vertices, if it comes up heads put an edge between the pair. The probability of pulling out a complete graph must be the same as the probability of generating a complete graph with this process.
4. Now what is the probability of generating a graph with order n and diameter greater than 2? Well, there would have to be a pair of vertices with no common neighbor. Fix vertices v and w. What is the probability that they have no common neighbor?
5. There are $\binom{n}{2}$ pairs of vertices in a graph with n vertices. What is the expected number of pairs with no common neighbor?
6. What can we conclude?

Notes

1. (**Lemma**) If $[S, T]$ is a source/sink cut, then the net flow of a flow f out of S equals the net flow into T. Furthermore, the net flow out of *any* source/sink cut is constant (and also equals the net flow out of s, and also equals $val(f)$.

2. (**Weak Duality**) If f is a feasible flow and [S,T] is a source/sink cut, then $val(f) \leq cap(S, T)$.

3. What is the *minimum cut* problem?

4. What is the *Ford-Fulkerson labeling algorithm*?

5. (**Max-flow Min-cut Theorem—AKA Ford-Fulkerson Theorem**) In every network, the maximum value of a feasible flow equals the minimum capacity of a source/sink cut.