Last name ______________________

First name ______________________

LARSON—MATH 656—CLASSROOM WORKSHEET 25
Network Flows.

Organizational Notes

1. Don't forget to send your Notes / Classroom worksheet after each class (make the email subject useful: like "Math 656 c25 notes").
2. The VCU Discrete Math Seminar is every Wednesday.
3. Read ahead! We're talking about Network Flow problems (Sec. 4.3)

Review

1. What is a *network*?

2. What is a *flow*? What is $f^+(v)$ and $f^-(v)$?

3. What is a *feasible flow*? What are *capacity constraints*?

4. What are *conservation constraints*?

5. What is the *value* $val(f)$ of a flow f? What is a *maximum flow*?

6. What is the *tolerance* of an edge e, of a path P? What is a f-augmenting path?

7. (**Lemma**) If P is an f-augmenting path with tolerance z then changing flow by $+z$ on edges followed forward by P and by $-z$ on edges followed backward by P produces a feasible flow f' with $vaf(f') = val(f) + z$.

8. What is a *source/sink cut [S,T]*? What is the *capacity*, $cap(S,T)$, of a cut [S,T]?

9. If $U \subseteq V(G)$, what is $f^+(U)$ and $f^-(U)$?

10. (**Lemma**) If U is a set of nodes in a network, then the net flow out of U is the sum of the net flows out of the nodes of U, that is,

$$f^+(U) - f^-(U) = \sum_{v \in U} [f^+(v) - f^-(v)].$$

Notes

1. (**Lemma**) If $[S, T]$ is a source/sink cut, then the net flow of a flow f out of S equals the net flow into T. Furthermore, the net flow out of *any* source/sink cut is constant (and also equals the net flow out of s, and also equals $val(f)$.

2. (**Weak Duality**) If f is a feasible flow and [S,T] is a source/sink cut, then $val(f) \leq cap(S, T)$.

3. What is the *minimum cut* problem?

4. What is the *Ford-Fulkerson labeling algorithm*?

5. (**Max-flow Min-cut Theorem—AKA Ford-Fulkerson Theorem**) In every network, the maximum value of a feasible flow equals the minimum capacity of a source/sink cut.