Last name _____

First name _____

LARSON—MATH 656—CLASSROOM WORKSHEET 22
Petersen's Theorem & Network Flows.

Organizational Notes

1. Don't forget to send your Notes / Classroom worksheet after each class (make the email subject useful: like "Math 656 c22 notes").
2. The VCU Discrete Math Seminar is every Wednesday.
3. $h07$ (the Gallai-Edmonds decomposition worksheet) is due today.
4. Read ahead! Next up we'll talk about Network Flow problems (Sec. 4.3)

Concepts & Notation

- Edmonds-Gallai Decomposition (West paper).
- Petersen's Theorem (Sec. 3.3).
- Network Flows (Sec. 4.3).

Review

1. (**Gallai-Edmonds Structure Theorem**). Let A, C, D, be the sets in the Gallai-Edmonds Decomposition of a graph G. Let $G_1,\ldots,G_k$ be the components of $G[D]$. If M is a maximum matching in G then:
   (a) M covers C and matches A into distinct components of $G[D]$.
   (b) Each G_i is factor-critical and M restricts to a near-perfect matching on G_i,
   (c) If $S \subseteq A$ is non-empty then $N_G(S)$ has a vertex in at least $|S|+1$ of $G_1,\ldots,G_k$,
   (d) $def(A) = def(G) = k - |A|$.

Notes

1. (**Petersen's Theorem**) If a graph has a perfect matching and no cut edges then it has a perfect matching.

2. What is a *directed graph*?

3. What is a *network*?

4. What is the *capacity* $c(e)$ of an edge e?

5. What are *source* and *sink* vertices?

6. What is a *flow*? What is $f^+(v)$ and $f^-(v)$?

7. What is a *feasible flow*? What are *capacity constraints*?

8. What are *conservation constraints*?

9. What is the *value* $val(f)$ of a flow f?

10. What is a *maximum flow*?

11. What is a f-augmenting path?

12. What is the *tolerance* of a flow?

13. (**Lemma**) If P is an f-augmenting path with tolerance z then changing flow by $+z$ on edges followed forward by P and by $-z$ on edges followed backward by P produces a feasible flow f' with $vaf(f') = val(f) + z$.