Last name ______________________

First name ______________________

LARSON—MATH 656—CLASSROOM WORKSHEET 19
Gallai-Edmonds-Decomposition.

Organizational Notes

1. Don't forget to send your Notes / Classroom worksheet after each class (make the email subject useful: like "Math 656 c19 notes").
2. The VCU Discrete Math Seminar is every Wednesday.
3. $h06$ is due on Wednesday (#3.3.1, 3.3.2, 3.3.3, 3.3.6, 3.3.10).
4. Read ahead! Next up we'll talk about Petersen's Theorem (Corollary to Tutte's Theorem in Sec 3.3) and then Network Flow problems (Sec. 4.3)

Concepts & Notation

- factor-critical graph, near-perfect matching, Edmonds-Gallai Decomposition (West paper).
- Petersen's Theorem (Sec. 3.3).
- Network Flows (Sec. 4.3).

Review

1. (**Theorem**) (Berge-Tutte Formula) $\nu = \frac{1}{2}(n - def(G))$.

2. (**Theorem**) (Tutte's Theorem) A graph G has a perfect matching if and only if for every $S \subseteq V(G)$ $o(G-S) \leq |S|$.

Notes

1. What is a *factor-critical graph*?
2. What is a *near-perfect* matching?
3. A vertex v in a graph is either (1) covered by every maximum matching (set B), or (2) not covered by every maximum matching (set D). A vertex in B either (1) has a neighbor outside B (set A) or (2) does not (set C). The **Gallai-Edmonds Decomposition** is the partition of $V(G)$ into sets C, A and D.
4. Find the Gallai-Edmonds Decomposition for a graph with a perfect matching.

5. Find the Gallai-Edmonds Decomposition for P_3.

6. Find the Gallai-Edmonds Decomposition for S_4.

7. Find the Gallai-Edmonds Decomposition for the house graph.

8. Find the Gallai-Edmonds Decomposition for the graph formed by the join of $3K_3$ and P_2.

9. (**Gallai-Edmonds Structure Theorem**). Let A, C, D, be the sets in the Gallai-Edmonds Decomposition of a graph G. Let $G_1,\ldots,G_k$ be the components of $G[D]$. If M is a maximum matching in G then:

   (a) M covers C and matches A into distinct components of $G[D]$.

   (b) Each G_i is factor-critical and M restricts to a near-perfect matching on G_i,

   (c) If $S \subseteq A$ is non-empty then $N_G(S)$ has a vertex in at least $|S|+1$ of $G_1,\ldots,G_k$,

   (d) $def(A) = def(G) = k - |A|$.

10. What does the Gallai-Edmonds Structure Theorem say for a graph with a perfect matching? Find a maximum matching M and check. Try some non-empty subsets $S \subseteq A$. Find $def(A)$, $def(G)$, k.

11. What does the Gallai-Edmonds Structure Theorem say for P_3? Find a maximum matching M and check. Try some non-empty subsets $S \subseteq A$. Find $def(A)$, $def(G)$, k.

12. What does the Gallai-Edmonds Structure Theorem say for S_4? Find a maximum matching M and check. Try some non-empty subsets $S \subseteq A$. Find $def(A)$, $def(G)$, k.

13. What does the Gallai-Edmonds Structure Theorem say for the house graph? Find a maximum matching M and check. Try some non-empty subsets $S \subseteq A$. Find $def(A)$, $def(G)$, k.

14. What does the Gallai-Edmonds Structure Theorem say for the graph formed by the join of $3K_3$ and P_2? Find a maximum matching M and check. Try some non-empty subsets $S \subseteq A$. Find $def(A)$, $def(G)$, k.