Last name _____

First name _____

LARSON—MATH 656—CLASSROOM WORKSHEET 16
Berge-Tutte Formula, Tutte's Theorem, Gallai-Edmonds-Decomposition.

Organizational Notes

1. Don't forget to send your Notes / Classroom worksheet after each class (make the email subject useful: like "Math 656 c15 notes").
2. The VCU Discrete Math Seminar is every Wednesday.
3. Homework #5 (h05) is the Test Review, due 11:59 p.m., Sunday, Mar. 21.
4. Test 1 is Monday, Mar. 22.
5. Read ahead! Next up we'll talk about Tutte's Theorem and the Gallai-Edmonds Mathcing Decomposition (as described in the West paper).

Concepts & Notation

- Sec. 3.3: general (cardinality) matching, Tutte's Theorem, Edmonds-Gallai Decomposition.

Review

1. What is the main idea of Edmond's Blossom Algorithm?

2. What is a k-factor in a graph? What is a 1-factor? (And what is the difference from a perfect matching?)

3. (**Notation**) Given a set $S \subseteq V(G)$, what is $o(G - S)$?

4. What is Tutte's Condition?

5. What is Tutte's Theorem?

Notes

1. (**Notation**) What is $def(S)$?

2. (**Notation**) What is $def(G)$?

3. What is the Berge-Tutte Formula?

4. **Claim** Any matching leaves at least $n - def(G)$ vertices unsaturated.

5. **Parity Lemma**: $o(G - S) - |S| \equiv n \pmod 2$.

6. **Maximal Maximum Deficiency Set Lemma** Let T be a maximal maximum deficiency set. Let u be a vertex of an odd component C of $G - T$. Then (1) $C - u$ satisfies Tutte's condition, and (2) the components of $G - u$ are all odd.

7. **Auxilliary Graph H(T)**. If T is a maximal maximum deficiency set, define the graph $H(T)$ with vertex set Y consisting of one vertex for each (odd) component of $G - T$, the vertices T and $y \in Y$ adjacent to $v \in T$ if any vertex in the component corresponding to y is adjacent to v. ($H(T)$ is a $T - Y$-bigraph).

8. **(**Lemma). $H(T)$ has a matching that covers T.

9. Theorem (**Berge-Tutte Formula**) $\nu = \frac{1}{2}(n - def(G))$.