CHAPTER 27

Implicit Differentiation

We now have a good working knowledge of derivatives. We know what they are, what they mean, and we have a list of rules for computing them. In this chapter we will not learn any more rules. Instead we will learn a *technique* that expands the scope of our existing rules. That technique is called *implicit differentiation*.

To explain *implicit* differentiation, we first review *regular* differentiation. Differentiation is a process we apply to *functions*. Given a function, say $f(x) = x^2 - 5$ (below, left), we can compute its derivative $f'(x) = 2x$, and $f'(x)$ equals the slope of the tangent to $y = f(x)$ at x.

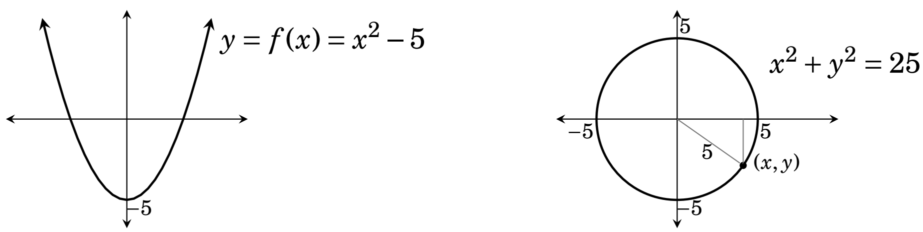


Figure 27.1. Left: Graph of a function. Right: Graph of an equation.

But what if instead we had the graph of an *equation*, like the graph of the equation $x^2 + y^2 = 25$, above. The graph of this equation is the circle of radius 5 centered at the origin. This is not the graph of a function because it fails the vertical line test. And because we can only take derivatives of *functions*, it makes no sense to talk about the derivative of this equation.

But still, we might reasonably ask what is the slope of a line that is tangent to the circle. By all rights, a *derivative* should give the answer. As we will see, implicit differentiation is a way of applying differentiation to equations that may not be functions.

The technique of implicit differentiation requires just one definition. Given an equation whose graph may fail the vertical line test, we can delete portions of the graph to achieve a graph that *does pass* the vertical line test, and therefore *is* the graph of a function. Any function obtained this way is called an **implicit function** of the original equation.

For example, Figure 27.2 shows the graph of the equation $x^2+y^2=25$ (which fails the vertical line test) and three implicit functions of it, each obtained by erasing parts of the graph of $x^2+y^2=25$.

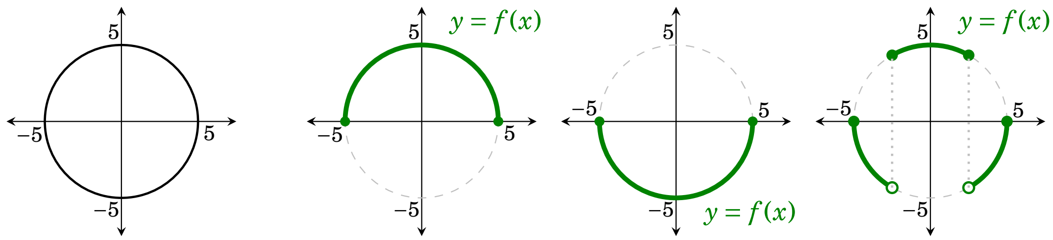
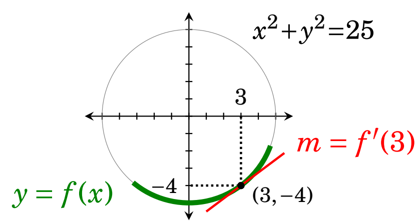


Figure 27.2. The graph of the equation $x^2+y^2=25$ (left) and the graphs of three implicit functions of this equation.

Motivational Problem We motivate and explain implicit differentiation with a simple example: Consider the graph of the equation $x^2+y^2=25$. The point $(3,-4)$ is on its graph because $3^2+(-4)^2=25$. **Problem:** find the slope of the tangent to the graph at $(3,-4)$.

Solution Envision an implicit function $f(x)$ of $x^2+y^2=25$ whose graph contains the point $(3,-4)$, as shown on the right. Then the slope of the tangent will be $f'(3)$. So we just need to find $f'(x)$ and plug in $x=3$.



Here is how we can find $f'(x)$. Start with our equation

$$x^2+y^2 = 25.$$

Since any point $(x, f(x))$ is on the circle, we can plug in $y=f(x)$ to get

$$x^2+\big(f(x)\big)^2 = 25.$$

Now both sides are functions of x alone. Differentiate both sides.

$$\begin{aligned} \frac{d}{dx}\Big[x^2+\big(f(x)\big)^2\Big] &= \frac{d}{dx}\Big[25\Big] \\ 2x+2f(x)f'(x) &= 0 \end{aligned}$$

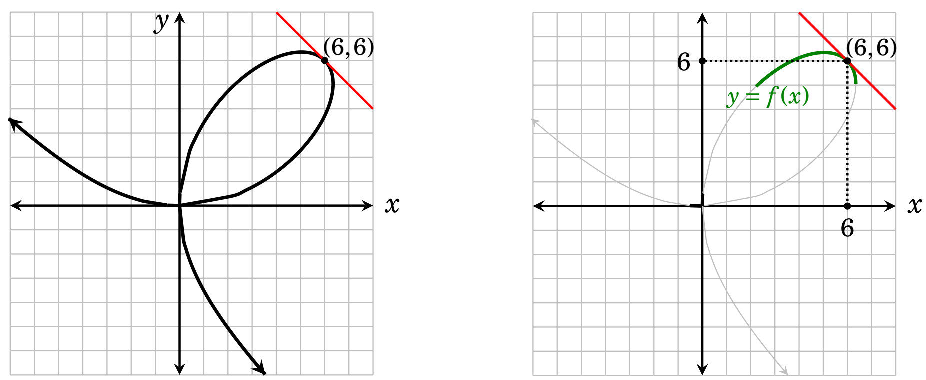
There's an $f'(x)$ in this equation. Since we are looking for $f'(x)$, solve for it.

$$\begin{aligned} 2f(x)f'(x) &= -2x \\ f'(x) &= -\frac{2x}{2f(x)} = -\frac{x}{f(x)} \end{aligned}$$

Answer: The tangent has slope $f'(3) = -\dfrac{3}{f(3)} = -\dfrac{3}{-4} = \dfrac{3}{4}$. ✍

We'll do another example and then codify our process into a technique.

Example 27.1 Consider $x^3 + y^3 = 12xy$. You may not be familiar with the graph of this equation, but it is sketched below. Notice that the point $(6,6)$ is on the graph because $6^3 + 6^3 = 12 \cdot 6 \cdot 6$. **Problem:** find the slope of the tangent at $(6,6)$. (From the picture you might guess that the slope is -1.)



Solution This equation is not a function, so imagine an implicit function $y = f(x)$ of it (above, right). In the equation, replace each y with $f(x)$:

$$\begin{aligned} x^3 + y^3 &= 12xy \\ x^3 + \big(f(x)\big)^3 &= 12xf(x) \end{aligned}$$

Both sides are now functions of x. Differentiate using whichever rules apply.

$$\begin{aligned} \frac{d}{dx}\Big[x^3 + \big(f(x)\big)^3\Big] &= \frac{d}{dx}\Big[12xf(x)\Big] \\ 3x^2 + 3\big(f(x)\big)^2 f'(x) &= 12f(x) + 12xf'(x) \end{aligned}$$

There are two occurrences of $f'(x)$ here, so we can solve for $f'(x)$. Get all terms with an $f'(x)$ on one side, factor out an $f'(x)$ and divide:

$$\begin{aligned} 3\big(f(x)\big)^2 f'(x) - 12xf'(x) &= 12f(x) - 3x^2 \\ f'(x)\Big(3\big(f(x)\big)^2 - 12x\Big) &= 12f(x) - 3x^2 \\ f'(x) &= \frac{12f(x) - 3x^2}{3\big(f(x)\big)^2 - 12x} \end{aligned}$$

This is $f'(x)$. Plug in $x=6$ to get the slope. (Note $f(6) = 6$, from the graph.)

Answer: Slope is $f'(6) = \dfrac{12f(6) - 3\cdot 6^2}{3\big(f(6)\big)^2 - 12\cdot 6} = \dfrac{12\cdot 6 - 3\cdot 6^2}{3\cdot 6^2 - 12\cdot 6} = -1$, as expected. ✍

The process used in our two examples is called *implicit differentiation*: Given an equation that is not a function (like $x^3+y^3=12xy$ in Example 27.1), replace y with an implicit function $y=f(x)$. Then the equation has just one variable x, and we can differentiate both sides. Doing this results in an equation involving the derivative $f'(x)=\frac{dy}{dx}$. We then solve for the derivative.

Actually, in applying this process we do not usually explicitly write $f(x)$ for each y. Instead we just *remember* that each y is really $f(x)$. This makes our writing more concise, but we must be careful. Do not mistakenly write, say, $\frac{d}{dx}\left[y^3\right]=3y^2$. This is wrong. Because $y=f(x)$, the $\frac{d}{dx}\left[y^3\right]$ is really $\frac{d}{dx}\left[\left(f(x)\right)^3\right]=3\left(f(x)\right)^2 f'(x)$ (by the generalized power rule). In practice, we'd write the computation from Example 27.1 as follows:

$$
\begin{aligned}
x^3+y^3 &= 12xy && \text{(don't forget: } y=f(x).\text{)}\\
\frac{d}{dx}\left[x^3+y^3\right] &= \frac{d}{dx}\left[12xy\right] && \text{(differentiate both sides.)}\\
3x^2+3y^2\frac{dy}{dx} &= 12y+12x\frac{dy}{dx} && \text{(now solve for } \tfrac{dy}{dx}.\text{)}\\
3y^2\frac{dy}{dx}-12x\frac{dy}{dx} &= 12y-3x^2 &&\\
\frac{dy}{dx}\left(3y^2-12x\right) &= 12y-3x^2 &&\\
\frac{dy}{dx} &= \frac{12y-3x^2}{3y^2-12x} && \text{(this is } \tfrac{dy}{dx}!\text{)}
\end{aligned}
$$

Now that we've found $\frac{dy}{dx}$, plug $(x,y)=(6,6)$ into it to get the slope at $(6,6)$.

$$\left.\frac{dy}{dx}\right|_{(x,y)=(6,6)} = \frac{12\cdot 6-3\cdot 6^2}{3\cdot 6^2-12\cdot 6} = -1.$$

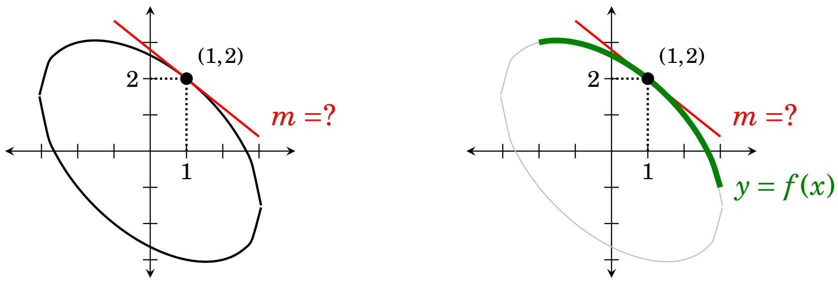
Here is a codification of the process that we have employed in the previous two examples.

Implicit Differentiation

Given an equation in x and y, how to find $\dfrac{dy}{dx}$:

1. Regard $y=f(x)$ as an implicit function of x.
2. Differentiate both sides of the equation with respect to x.
3. Solve for $\dfrac{dy}{dx}$.
4. Plug in coordinates (e.g. to get slope) if necessary.

Example 27.2 The graph of the equation $x^2 + y^2 = 7 - xy$ is an ellipse, as indicated below, left. Note that the point $(1,2)$ is on the graph of the equation. Find the slope of the tangent line to the graph at $(1,2)$.



Solution The graph of $x^2 + y^2 = 7 - xy$ fails the vertical line test, so we do not have a function. To overcome this deficiency, erase part of the graph, leaving an implicit function $y = f(x)$ whose graph passes through $(1,2)$. Keeping in mind that $y = f(x)$, we apply steps 1–4 of implicit differentiation.

$$\begin{aligned}
x^2 + y^2 &= 7 - xy \\
\frac{d}{dx}\left[x^2 + y^2\right] &= \frac{d}{dx}\left[7 - xy\right] \\
2x + 2y^1\frac{dy}{dx} &= 0 - 1\cdot y - x\frac{dy}{dx} && \text{(Remember: } y = f(x)) \\
2y\frac{dy}{dx} + x\frac{dy}{dx} &= -y - 2x \\
\frac{dy}{dx}\left(2y + x\right) &= -y - 2x \\
\frac{dy}{dx} &= \frac{-y - 2x}{2y + x}
\end{aligned}$$

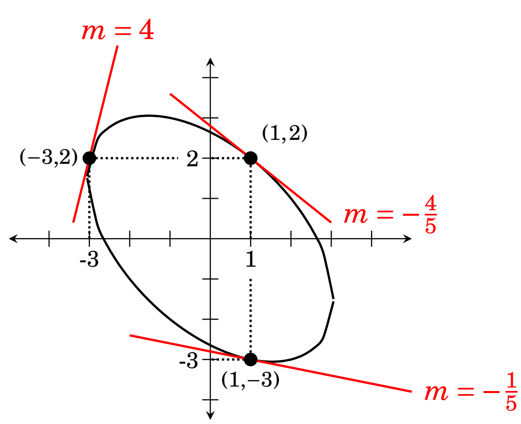
Answer: The slope of the tangent at $(1,2)$ is $\left.\frac{dy}{dx}\right|_{(x,y)=(1,2)} = \frac{-2-2\cdot 1}{2\cdot 2+1} = \boxed{-\frac{4}{5}.}$

It's important to note that once we've found $\frac{dy}{dx}$, we can use it to get the slope an *any point on the curve:*

$$\left.\frac{dy}{dx}\right|_{(x,y)=(1,-3)} = \frac{-(-3)-2\cdot 1}{2\cdot(-3)+1} = \boxed{-\frac{1}{5}.}$$

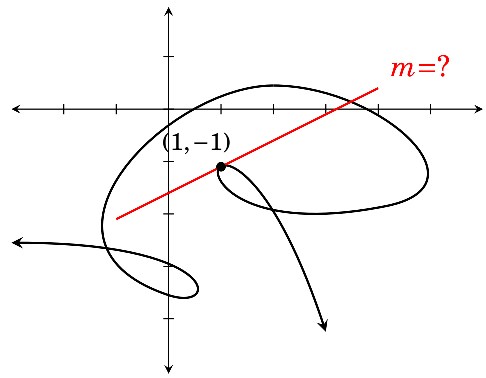
$$\left.\frac{dy}{dx}\right|_{(x,y)=(-3,2)} = \frac{-2-2\cdot(-3)}{2\cdot 2+(-3)} = \boxed{4.}$$

The derivative is a machine that spits out slope at any point. You just have to plug in the coordinates. ✍



Example 27.3 Consider the equation $\ln|xy| = y+1$. Find $\frac{dy}{dx}$. Then find the slope of the tangent line to the graph of the equation at the point $(1,-1)$.

Solution It's hard to say what the graph of this equation looks like, though it does pass through the point $(1,-1)$, since $(x,y) = (1,-1)$ satisfies the equation. For all we know, the graph could look something like the generic curve shown on the right. Fortunately we don't need the graph to solve the problem. The question is not what the graph looks like, but what is $\frac{dy}{dx}$.



To find $\frac{dy}{dx}$ we just need to differentiate the equation implicitly. To streamline our writing, below we use y' to denote $\frac{dy}{dx}$.

$$\begin{aligned}
\ln|xy| &= y+1 \\
\frac{d}{dx}\Big[\ln|xy|\Big] &= \frac{d}{dx}\Big[y+1\Big] \\
\frac{1}{xy}\frac{d}{dx}\Big[xy\Big] &= \frac{d}{dx}\Big[y\Big]+\frac{d}{dx}\Big[1\Big] \\
\frac{1}{xy}\left(1\cdot y+xy'\right) &= y'+0 && \text{(now solve for } y'\text{)} \\
y+xy' &= xyy' && \text{(multiply both sides by } xy\text{)} \\
y &= xyy'-xy' \\
y &= y'(xy-x) \\
\frac{y}{xy-x} &= y' && (y' \text{ is the derivative!})
\end{aligned}$$

Answer The derivative is $\dfrac{dy}{dx} = \dfrac{y}{xy-x}$. The slope of the tangent at $(1,-1)$ is $\dfrac{dy}{dx}\Big|_{(x,y)=(1,-1)} = \dfrac{-1}{1\cdot(-1)-1} = \boxed{\dfrac{1}{2}}.$ ✍

Occasionally you may need to compute a *second* derivative $\frac{d^2y}{dx^2}$ (or perhaps an even higher derivative) with implicit differentiation. The next example addresses this.

Example 27.4 Given the equation $\ln|xy| = y+1$, find $\frac{d^2y}{dx^2}$.

Solution In Example 27.3 we found that $\frac{dy}{dx} = \frac{y}{xy-x}$, so we need to take the derivative of this. But $\frac{y}{xy-x}$ is not a function of x alone, as it has both x's and y's in it. The key to overcoming this is—as in Example 27.3—is to view y as an implicit function of x. Every time you see a y, remember $y = f(x)$.

$$\begin{aligned}
\frac{d^2y}{dx^2} &= \frac{d}{dx}\left[\frac{dy}{dx}\right] \\
&= \frac{d}{dx}\left[\frac{y}{xy-x}\right] \\
&= \frac{\frac{d}{dx}\left[y\right]\cdot(xy-x) - y\cdot\frac{d}{dx}\left[xy-x\right]}{(xy-x)^2} && \text{(quotient rule)} \\
&= \frac{\frac{d}{dx}\left[y\right]\cdot(xy-x) - y\left(\frac{d}{dx}\left[xy\right] - \frac{d}{dx}\left[x\right]\right)}{(xy-x)^2} && \text{(sum-diff. rule)} \\
&= \frac{\frac{d}{dx}\left[y\right]\cdot(xy-x) - y\left(1\cdot y + x\cdot\frac{d}{dx}\left[y\right] - 1\right)}{(xy-x)^2} && \text{(product rule)}
\end{aligned}$$

At this point (or even *before* this point), remember that in Example 27.3 we found $\frac{dy}{dx} = \frac{y}{xy-x}$, that is, $\frac{d}{dx}[y] = \frac{y}{xy-x}$. Make this replacement:

$$= \frac{\frac{y}{xy-x}\cdot(xy-x) - y\left(y + x\cdot\frac{y}{xy-x} - 1\right)}{(xy-x)^2}.$$

This is $\frac{d^2y}{dx^2}$! But it can be simplified somewhat. Continuing our work,

$$= \frac{y - y\left(y + \frac{xy}{xy-x} - 1\right)}{(xy-x)^2} = \frac{y - y\left(y + \frac{y}{y-1} - 1\right)}{(xy-x)^2}$$

$$= \frac{2y - y^2 - \frac{y^2}{y-1}}{(xy-x)^2} = \frac{2y-y^2}{(xy-x)^2} - \frac{y^2}{(y-1)(xy-x)^2}.$$

Answer The second derivative is $\frac{d^2y}{dx^2} = \frac{2y-y^2}{(xy-x)^2} - \frac{y^2}{(y-1)(xy-x)^2}$. ✍

Implicit differentiation problems can play out in a great variety of ways. It's important to get ample practice.

Exercises for Chapter 27

1. Consider the equation $x^2 + xy - y^2 = 1$. Use implicit differentiation to find $\frac{dy}{dx}$. Then find the slope of the tangent line to the graph at the point $(2,3)$.
2. Consider the equation $x^2y^2 = 9$. Use implicit differentiation to find $\frac{dy}{dx}$. Then find the slope of the tangent line to the graph at the point $(-1,3)$.
3. Consider the equation $xy^3 = xy + 6$. Use implicit differentiation to find $\frac{dy}{dx}$. Then find the **equation of the tangent line** to the graph at the point $(1,2)$.
4. Consider the equation $x^2 + xy + y^2 = 7$. Use implicit differentiation to find $\frac{dy}{dx}$. Then find the **equation of the tangent line** to the graph at the point $(2,-3)$.
5. Consider the equation $\sin(xy^3) = y$. Use implicit differentiation to find $\frac{dy}{dx}$.
6. Consider the equation $x\cos(y^3) = e^y$. Use implicit differentiation to find $\frac{dy}{dx}$.
7. Consider the equation $x\sin(y) = y^3$. Use implicit differentiation to find $\frac{dy}{dx}$.
8. Consider the equation $x\tan(y^3) = y$. Use implicit differentiation to find $\frac{dy}{dx}$.
9. Consider the equation $e^y = 2\cos(2x)$. Use implicit differentiation to find $\frac{dy}{dx}$. Then find the slope of the tangent line to the graph at the point $\left(\frac{\pi}{6}, 0\right)$.
10. Consider the equation $e^x = 2\cos(2y)$. Use implicit differentiation to find $\frac{dy}{dx}$. Then find the slope of the tangent line to the graph at the point $\left(0, \frac{\pi}{6}\right)$.
11. Consider the equation $y\cos(y) = x^2$. Use implicit differentiation to find $\frac{dy}{dx}$. Then find the slope of the tangent line to the graph at the point $(\sqrt{\pi}, -\pi)$.
12. Consider the equation $x\sin(y) = y$. Use implicit differentiation to find $\frac{dy}{dx}$. Then find the slope of the tangent line to the graph at the point $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$.
13. Consider the equation $2xy + \pi\sin(y) = 2\pi$. Use implicit differentiation to find $\frac{dy}{dx}$. Then find the slope of the tangent line to the graph at the point $(1, \pi/2)$.
14. Consider the equation $y = 2\sin(\pi x - y)$. Use implicit differentiation to find $\frac{dy}{dx}$. Then find the slope of the tangent line to the graph at the point $(1,0)$.
15. Consider the equation $x^3 + y^3 = 4xy$. Use implicit differentiation to find $\frac{dy}{dx}$. Then find the slope of the tangent line to the graph of the equation at the point $(2,2)$.
16. Consider the equation $\cos(y^2) + x = e^y$. Use implicit differentiation to find $\frac{dy}{dx}$. Then find the slope of the tangent line to the graph of the equation at $(0,0)$.
17. Consider the equation $\cos(y) + x^2 + x = e^y$. Use implicit differentiation to find $\frac{dy}{dx}$. Then find the slope of the tangent line to the graph of the equation at $(0,0)$.
18. Consider the equation $x\sqrt[3]{y}^2 + y = 12$. Use implicit differentiation to find $\frac{dy}{dx}$. Then find the slope of the tangent line to the graph of the equation at the point $(1,8)$.

Exercise Solutions for Chapter 27

1. Consider the equation $x^2 + xy - y^2 = 1$. Use implicit differentiation to find $\frac{dy}{dx}$. Then find the slope of the tangent line to the graph at the point $(2,3)$.

$$\begin{aligned}
\frac{d}{dx}\left[x^2 + xy - y^2\right] &= \frac{d}{dx}\left[1\right] \\
2x + 1\cdot y + xy' - 2yy' &= 0 \\
xy' - 2yy' &= -2x - y \\
y'(x-2y) &= -2x - y \\
y' &= \frac{-2x - y}{x - 2y}
\end{aligned}$$

The slope at $(2,3)$ is $y'\Big|_{(x,y)=(2,3)} = \frac{-2\cdot 2 - 3}{2 - 2\cdot 3} = \frac{7}{4}$.

3. Consider the equation $xy^3 = xy + 6$. Use implicit differentiation to find $\frac{dy}{dx}$. Then find the **equation of the tangent line** to the graph at the point $(1,2)$.

$$\begin{aligned}
\frac{d}{dx}\left[xy^3\right] &= \frac{d}{dx}\left[xy + 6\right] \\
1\cdot y^3 + x3y^2y' &= 1\cdot y + xy' + 0 \\
3xy^2y' - xy' &= y - y^3 \\
y'\left(3xy^2 - x\right) &= y - y^3 \\
y' &= \frac{y - y^3}{3xy^2 - x}
\end{aligned}$$

The slope at $(1,2)$ is $y'\Big|_{(x,y)=(1,2)} = \frac{2 - 2^3}{3\cdot 1\cdot 2^2 - 1} = -\frac{6}{11}$. We can get its equation with the point-slope formula for a line:

$$\begin{aligned}
y - y_0 &= m(x - x_0) \\
y - 2 &= -\frac{6}{11}(x-1) \\
y &= -\frac{6}{11}x + \frac{6}{11} + 2 \\
y &= -\frac{6}{11}x + \frac{28}{11}.
\end{aligned}$$

5. Consider the equation $\sin(xy^3) = y$. Use implicit differentiation to find $\frac{dy}{dx}$.

$$\begin{aligned}
\frac{d}{dx}\left[\sin\left(xy^3\right)\right] &= \frac{d}{dx}\left[y\right] \\
\cos\left(xy^3\right)\left(1\cdot y^3 + x\cdot 3y^2y'\right) &= y' \\
y^3\cos\left(xy^3\right) + 3xy^2y'\cos\left(xy^3\right) &= y' \\
3xy^2y'\cos\left(xy^3\right) - y' &= -y^3\cos\left(xy^3\right)
\end{aligned}$$

$$\begin{aligned} y'\left(3xy^2\cos\left(xy^3\right)-1\right) &= -y^3\cos\left(xy^3\right) \\ y' &= -\frac{y^3\cos\left(xy^3\right)}{3xy^2\cos\left(xy^3\right)-1} \end{aligned}$$

7. Consider the equation $x\sin(y) = y^3$. Use implicit differentiation to find $\frac{dy}{dx}$.

$$\begin{aligned} \frac{d}{dx}\left[x\sin(y)\right] &= \frac{d}{dx}\left[y^3\right] \\ 1\cdot\sin(y)+x\cos(y)y' &= 3y^2y' \\ x\cos(y)y'-3y^2y' &= -\sin(y) \\ y'\left(x\cos(y)-3y^2\right) &= -\sin(y) \\ y' &= \frac{-\sin(y)}{x\cos(y)-3y^2} \end{aligned}$$

9. Consider the equation $e^y = 2\cos(2x)$. Use implicit differentiation to find $\frac{dy}{dx}$. Then find the slope of the tangent line to the graph at the point $(\pi/6, 0)$.

$$\begin{aligned} \frac{d}{dx}\left[e^y\right] &= \frac{d}{dx}\left[2\cos(2x)\right] \\ e^y y' &= -2\sin(2x)\cdot 2 \\ y' &= -\frac{4\sin(2x)}{e^y} \end{aligned}$$

The slope at $(\pi/6, 0)$ is $y'\Big|_{(x,y)=(\pi/6,0)} = -\frac{4\sin(2\cdot\pi/6)}{e^0} = -\frac{4\sin(\pi/3)}{1} = -2\sqrt{3}$.

11. Consider the equation $y\cos(y) = x^2$. Use implicit differentiation to find $\frac{dy}{dx}$. Then find the slope of the tangent line to the graph at the point $\left(\sqrt{\pi}, -\pi\right)$.

$$\begin{aligned} \frac{d}{dx}\left[y\cos(y)\right] &= \frac{d}{dx}\left[x^2\right] \\ y'\cos(y)+y\left(-\sin(y)y'\right) &= 2x \\ y'\left(\cos(y)-y\sin(y)\right) &= 2x \\ y' &= \frac{2x}{\cos(y)-y\sin(y)} \end{aligned}$$

The slope at $(\sqrt{\pi}, -\pi)$ is $y'\Big|_{(x,y)=(\sqrt{\pi},-\pi)} = \frac{2\sqrt{\pi}}{\cos(-\pi)-(-\pi)\sin(-\pi)} = \frac{2\sqrt{\pi}}{-1-0} = -2\sqrt{\pi}$.

13. Consider the equation $2xy + \pi\sin(y) = 2\pi$. Use implicit differentiation to find $\frac{dy}{dx}$. Then find the slope of the tangent line to the graph at the point $(1, \pi/2)$.

$$\begin{aligned} \frac{d}{dx}\left[2xy+\pi\sin(y)\right] &= \frac{d}{dx}\left[2\pi\right] \\ 2y+2xy'+\pi\cos(y)y' &= 0 \\ 2xy'+\pi\cos(y)y' &= -2y \end{aligned}$$

$$\begin{aligned} y'(2x+\pi\cos(y)) &= -2y \\ y' &= \frac{-2y}{2x+\pi\cos(y)} \end{aligned}$$

The slope at $(1,\pi/2)$ is $y'\Big|_{(x,y)=(1,\pi/2)} = \dfrac{-2\pi/2}{2\cdot 1+\pi\cos(\pi/2)} = \dfrac{-\pi}{2+\pi\cdot 0} = -\dfrac{\pi}{2}$.

15. Consider the equation $x^3+y^3=4xy$. Use implicit differentiation to find $\frac{dy}{dx}$. Then find the slope of the tangent line to the graph of the equation at the point $(2,2)$.

$$\begin{aligned} \frac{d}{dx}\left[x^3+y^3\right] &= \frac{d}{dx}\left[4xy\right] \\ 3x^2+3y^2y' &= 4y+4xy' \\ 3y^2y'-4xy' &= 4y-3x^2 \\ y'\left(3y^2-4x\right) &= 4y-3x^2 \\ y' &= \frac{4y-3x^2}{3y^2-4x} \end{aligned}$$

The slope of the tangent at $(2,2)$ is $y'\Big|_{(x,y)=(2,2)} = \dfrac{4\cdot 2-3\cdot 2^2}{3\cdot 2^2-4\cdot 2} = -1$.

17. Consider the equation $\cos(y)+x^2+x=e^y$. Use implicit differentiation to find $\frac{dy}{dx}$. Then find the slope of the tangent line to the graph of the equation at $(0,0)$.

$$\begin{aligned} \frac{d}{dx}\left[\cos(y)+x^2+x\right] &= \frac{d}{dx}\left[e^y\right] \\ -\sin(y)y'+2x+1 &= e^y y' \\ e^y y'+\sin(y)y' &= 2x+1 \\ y'\left(e^y+\sin(y)\right) &= 2x+1 \\ y' &= \frac{2x+1}{e^y+\sin(y)} \end{aligned}$$

The slope at $(0,0)$ is $y'\Big|_{(x,y)=(0,0)} = \dfrac{2\cdot 0+1}{e^0+\sin(0)} = \dfrac{1}{1+0} = 1$.