One-Sample Hypothesis Test Examples  
(Chapter 10)

A certain soft drink bottler claims that less than 20% of its customers drink another brand of soft drink on a regular basis. A random sample of 100 customers yielded 18 who did in fact drink another brand of soft drink on a regular basis. Do these sample results support the bottler’s claim? (Use a level of significance of 0.05.)

**Step 0:** Check Assumptions  
\[ n\pi_0 = 100(0.20) = 20 \geq 10 \quad \text{and} \quad n(1 - \pi_0) = 100(0.80) = 80 \geq 10 \]

**Step 1:** Hypotheses  
- \( H_0: \pi = 20\% \)  
- \( H_a: \pi < 20\% \)  
- \( H_0: \pi = 0.20 \)  
- \( H_a: \pi < 0.20 \)

**Step 2:** Significance Level  
\( \alpha = 0.05 \)

**Step 3:** Critical Value(s) and Rejection Region(s)  
- \( z_{\alpha} = -z_{0.05} = -1.645 \)  
- Reject the null hypothesis if \( Z \leq -1.645 \).

**Step 4:** Test Statistic  
\[
Z = \frac{p - \pi_0}{\sqrt{\frac{\pi_0(1 - \pi_0)}{n}}} = \frac{18}{100} - 0.20 = -0.5 \quad p\text{-value} = P(Z \leq -0.5) = 0.3085
\]

**Step 5:** Conclusion  
Since \(-0.5 > -1.645\) (\(p\)-value = 0.3085 > 0.05 = \(\alpha\)), we fail to reject the null hypothesis.

**Step 6:** State conclusion in words  
At the \(\alpha = 0.05\) level of significance, there is not enough evidence to conclude that less than 20% of the customers drink another brand. Thus the results do not support the bottler’s claim.
2. A standardized test for a specific college course is constructed so that the distribution of grades should have $\mu = 100$ and $\sigma = 10$. A class of 30 students has a mean grade of 92.

a. Test the null hypothesis that the grades from this class are a random sample from the stated distribution. (Use $\alpha = 0.05$.)

**Step 1:** Hypotheses

$H_0$: $\mu = 100$

$H_a$: $\mu \neq 100$

**Step 2:** Significance Level

$\alpha = 0.05$

**Step 3:** Critical Value(s) and Rejection Region(s)

$\pm z_{\alpha/2} = \pm z_{0.025} = \pm 1.96$ Reject the null hypothesis if $Z \leq -1.96$ or $Z \geq 1.96$.

**Step 4:** Test Statistic

$Z = \frac{\bar{y} - \mu_0}{\sigma/\sqrt{n}} = \frac{92 - 100}{10/\sqrt{30}} = -4.3818$

$p$-value $= 2 \cdot P(z \geq 4.38) = 0.0000$

**Step 5:** Conclusion

Since $-4.3818 \leq -1.96$ ($p$-value $\approx 0.0000 \leq 0.05 = \alpha$), we shall reject the null hypothesis.

**Step 6:** State conclusion in words

At the $\alpha = 0.05$ level of significance, there is enough evidence to conclude that the average test score is not 100.

b. What is the $p$-value associated with this test?

$p$-value $= P(z \leq -4.38) + P(z \geq 4.38) = 2 \cdot P(z \geq 3.99) = 0.0000$ ($p$-value $= 0.00006334$)

c. Discuss the practical uses of the results of this statistical test.

This test has several practical uses. It can be used to determine the validity of the test if the group of students are deemed to be a random sample. It also can be used to determine if the group of students in the sample is similar to the entire population upon which the test is based.
3. A drug company is testing a drug intended to increase heart rate. A sample of 100 yielded a mean increase of 1.4 beats per minute, with a population standard deviation known to be 3.6. Since the company wants to avoid marketing an ineffective drug, it proposes a 0.001 significance level. Should it market the drug? (Hint: If the drug doesn’t work, increase will be zero.)

**Step 1:** Hypotheses

- $H_0: \mu = 0$ bpm
- $H_a: \mu > 0$ bpm

**Step 2:** Significance Level

$\alpha = 0.001$

**Step 3:** Critical Value(s) and Rejection Region(s)

$z_{cr} = z_{0.001} =$ 3.09 Reject the null hypothesis if $Z \geq 3.09$.

**Step 4:** Test Statistic and $P$-value

$$Z = \frac{\bar{y} - \mu_0}{\sigma / \sqrt{n}} = \frac{1.4 - 0}{3.6 / \sqrt{100}} = 3.8889 \quad p\text{-value} = P(Z \geq 3.89) \approx 0.0000$$

**Step 5:** Conclusion

Since $3.8889 \geq 3.09$ ($p$-value $\approx 0.0000 \leq 0.001 = \alpha$), we shall reject the null hypothesis.

**Step 6:** State conclusion in words

At the $\alpha = 0.001$ level of significance, there exists enough evidence to conclude that the drug increases heart rate. Therefore, the company should market this drug.

4. A manufacturer of auto windows has developed a new plastic protective material that can be applied much thinner than the conventional material. To use this material, however, the production machinery must be adjusted. A trial adjustment was made on one of the 10 machines used in production, and a sample of 25 windshields measured. This sample had a mean thickness of 2.9 mm. Using the population standard deviation of 0.25 mm, does this adjustment provide for a smaller thickness in the material than the old adjustment (4 mm)? (Use a hypothesis test and level of significance of 0.01. Assume the distribution of thickness is approximately normal.)

**Step 1:** Hypotheses

- $H_0: \mu = 4$ mm
- $H_a: \mu < 4$ mm

**Step 2:** Significance Level

$\alpha = 0.01$

**Step 3:** Critical Value(s) and Rejection Region(s)

$-z_{cr} = -z_{0.01} = -2.33$ Reject the null hypothesis if $Z \leq -2.33$.

**Step 4:** Test Statistic and $P$-value

$$Z = \frac{\bar{y} - \mu_0}{\sigma / \sqrt{n}} = \frac{2.9 - 4}{0.25 / \sqrt{25}} = -22 \quad p\text{-value} = P(Z \leq -22) = 0.0000$$

**Step 5:** Conclusion

Since $-22 \leq -2.33$ ($p$-value $\approx 0.0000 \leq 0.01 = \alpha$), we shall reject the null hypothesis.

**Step 6:** State conclusion in words

At the $\alpha = 0.01$ level of significance, there exists enough evidence to conclude that this adjustment provides for a thickness smaller than 4 mm.
5. The manufacturer in Example 3 tried another, less expensive adjustment on another machine. A sample of 25 windshields was measured yielding a sample mean thickness of 3.4. Calculate the \( p \)-value resulting from this mean using the same hypothesis and assumptions as in Example 3.

**Step 1:** Hypotheses

\( H_0: \mu = 4 \text{ mm} \)

\( H_a: \mu < 4 \text{ mm} \)

**Step 2:** Significance Level

\( \alpha = 0.01 \)

**Step 3:** Critical Value(s) and Rejection Region(s)

\(-z_{\alpha} = -z_{0.01} = -2.33 \) \( \text{ Reject the null hypothesis if } Z \leq -2.33. \)

**Step 4:** Test Statistic and \( P \)-value

\[ Z = \frac{\bar{y} - \mu_0}{\sigma \sqrt{n}} = \frac{3.4 - 4}{0.25 \sqrt{25}} = -12 \]

\( p \)-value \( = P(Z \leq -12) \approx 0.0000 \)

**Step 5:** Conclusion

Since \(-12 \leq -2.33 \) (\( p \)-value \( \approx 0.0000 \leq 0.01 = \alpha \)), we'll reject the null hypothesis.

**Step 6:** State conclusion in words

At the \( \alpha = 0.01 \) level of significance, there exists enough evidence to conclude that this adjustment provides for a thickness smaller than 4 mm.

6. Weight losses of 12 persons in an experimental one-week diet program are given below:

<table>
<thead>
<tr>
<th>Weight Loss in Pounds</th>
<th>3.0</th>
<th>1.4</th>
<th>0.2</th>
<th>-1.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.3</td>
<td>1.7</td>
<td>3.7</td>
<td>5.9</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>3.6</td>
<td>3.7</td>
<td>2.0</td>
<td></td>
</tr>
</tbody>
</table>

Do these results indicate that a mean weight loss was achieved? (Assume that the population is normally distributed and use \( \alpha = 0.05 \)).

**Step 1:** Hypotheses

\( H_0: \mu = 0 \text{ pounds} \)

\( H_a: \mu > 0 \text{ pounds} \)

**Step 2:** Significance Level

\( \alpha = 0.05 \)

**Step 3:** Critical Value(s) and Rejection Region(s)

\( t_{\alpha, df} = t_{0.05, df = 11} = 1.80 \) \( \text{ Reject the null hypothesis if } T \geq 1.80. \)

**Step 4:** Test Statistic

\[ \bar{y} = \frac{\sum y}{n} = 29.5 \]

\[ s = \sqrt{\frac{\sum y^2 - (\sum y)^2}{n - 1}} \]

\[ T = \frac{\bar{y} - \mu_0}{s \sqrt{n}} = \frac{2.4583 - 0}{2.1339 \sqrt{12}} = 3.9907 \]

\( p \)-value \( = P(T \geq 3.9907) \approx P(T \geq 4.0) = 0.001 \)

**Step 5:** Conclusion

Since \( 3.9907 \geq 1.80 \) (\( p \)-value \( = 0.001 \leq 0.05 = \alpha \)), we shall reject the null hypothesis.

**Step 6:** State conclusion in words

At the \( \alpha = 0.05 \) level of significance, there exists enough evidence to conclude that there is an average weight loss under this new diet.
7. In Exercise 6, determine if a mean weight loss of more than 1 pound was achieved. (Use $\alpha = 0.01$.)

**Step 1:** Hypotheses
- $H_0: \mu = 1 \text{ pounds}$
- $H_a: \mu > 1 \text{ pounds}$

**Step 2:** Significance Level
- $\alpha = 0.01$

**Step 3:** Critical Value(s) and Rejection Region(s)
- $t_{\alpha, df = n-1} = t_{0.01, df = 11} = 2.72$ Reject the null hypothesis if $T \geq 2.72$.

**Step 4:** Test Statistic
- $T = \frac{\bar{y} - \mu_0}{s / \sqrt{n}} = \frac{2.4583 - 1}{2.1339 / \sqrt{12}} = 2.3672$
- $p$-value = $P(t \geq 2.3672) = P(t \geq 2.4) = 0.018$

**Step 5:** Conclusion
Since $2.3672 < 2.72$ ($p$-value $= 0.018 > 0.01 = \alpha$), we fail to reject the null hypothesis.

**Step 6:** State conclusion in words
At the $\alpha = 0.01$ level of significance, there is not enough evidence to conclude that the average weight loss is more than one pound.

8. Average systolic blood pressure of a normal male is supposed to be about 129. Measurements of systolic blood pressure on a sample of 12 adult males from a community whose dietary habits are suspected of causing high blood pressure are listed below:

<table>
<thead>
<tr>
<th>115</th>
<th>134</th>
<th>131</th>
<th>143</th>
</tr>
</thead>
<tbody>
<tr>
<td>130</td>
<td>154</td>
<td>119</td>
<td>137</td>
</tr>
<tr>
<td>155</td>
<td>130</td>
<td>110</td>
<td>138</td>
</tr>
</tbody>
</table>

Do the data justify ($\alpha = 0.01$) the suspicions regarding the blood pressure of this community?

**Step 1:** Hypotheses
- $H_0: \mu = 129$
- $H_a: \mu > 129$

**Step 2:** Significance Level
- $\alpha = 0.01$

**Step 3:** Critical Value(s) and Rejection Region(s)
- $t_{\alpha, df = n-1} = t_{0.01, df = 11} = 2.72$ Reject the null hypothesis if $T \geq 2.72$.

**Step 4:** Test Statistic
- $\bar{y} = \frac{\sum y}{n} = \frac{1596}{12} = 133$
- $s = \sqrt{\frac{\sum y^2 - (\sum y)^2}{n - 1}} = \sqrt{\frac{214,406 - (1596)^2}{12}} = 13.9414$
- $T = \frac{\bar{y} - \mu_0}{s / \sqrt{n}} = \frac{133 - 129}{13.9414 / \sqrt{12}} = 0.9939$
- $p$-value = $P(t \geq 0.9937) = P(t \geq 1.0) = 0.169$

**Step 5:** Conclusion
Since $0.9939 < 2.72$ ($p$-value $= 0.169 > 0.01 = \alpha$), we fail to reject the null hypothesis.

**Step 6:** State conclusion in words
At the $\alpha = 0.01$ level of significance, there is not enough evidence to conclude that there is high blood pressure in this community’s males.