Exercise 18

Please attempt all of the following problems before the due date. Your grade on this assignment will be calculated from the best three answers.

Problem 18.1

The text claims that, in the relation

\[ vf = v \cdot df \]

the right side of the equation is locally linear in the form \( df \) but the left side is not locally linear in the function \( f \). On the left side, \( v \) obeys Leibniz’s rule. On the right side, it does not. Multiply the function \( f \) by another function \( g \) and show that these properties are consistent. What is it that obeys Leibniz’s rule on the right side of the relation?

Answer 18.1

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

Problem 18.2

Show that the direct construction of the covariant derivative of a tensor by subtracting counter terms for each argument as in the expression,

\[
(D_v T)(P, a_1 (P), a_2 (P), ..., a_q (P)) = D_v (T (P, a_1 (P), a_2 (P), ..., a_q (P))) \\
- T (P, D_v a_1 (P), a_2 (P), ..., a_q (P)) \\
- T (P, a_1 (P), D_v a_2 (P), ..., a_q (P)) \\
\vdots \\
- T (P, a_1 (P), a_2 (P), ..., D_v a_q (P))
\]

does, in fact, produce an expression that is locally linear in each of its tensor arguments.

(Hint: Use the local linearity of \( T \) together with Leibniz’s product rule for \( L_v \) acting on the product of two functions and for \( D_v \) acting on the product of a function and a tensor.)

Answer 18.2

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.
Problem 18.3

Show that the direct construction procedure described above will always give the same result as assuming that tensor products and dot products obey Leibniz’ rule.
(Hint: Show that it works for vector and form fields and then use induction.)

Answer 18.3

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

Problem 18.4

Use the direct construction of the covariant derivative of a one-form field \( \alpha \)

\[
(D_v \alpha)(u) = \nabla_v (\alpha(u)) - \alpha(\nabla_v u)
\]

to obtain the result

\[
D_v \alpha = v^d \left( e_d (\alpha_A) - \alpha_K \Gamma^K_{A\ell} \right) W^A
\]

obtained by applying Leibniz’s rule to the dot product.

Answer 18.4

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.