Exercise 17

Please attempt all of the following problems before the due date. Your grade on this assignment will be calculated from the best three answers.

Problem 17.1

Show that the Lie derivative of a vector field $u$ obeys Leibniz’s product rule

$$\mathcal{L}_w fu = f \mathcal{L}_w u + (\mathcal{L}_w f) u$$

for any function $f$.  
(Note: You did this earlier. Only the notation has changed.)

Answer 17.1

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

Problem 17.2

Show that the ‘obvious’ way for one directional derivative to differentiate another,

$$\mathcal{L}_w u = wu,$$

does not yield a vector field.

Answer 17.2

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

Problem 17.3

Show that when the basis vectors are holonomic ($e_i = \frac{\partial}{\partial x^i}$), the Lie derivative just corresponds to differentiating the components of a vector field. In particular, show that

$$\mathcal{L}_e_i u = \left( \frac{\partial u^j}{\partial x^i} \right) e_j$$

Answer 17.3
Problem 17.4

Evaluate $L_{gw}u$ for vector fields $u, w$ and an arbitrary function $g$ and compare the result to $gL_{w}u$.

Answer 17.4

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.