Exercise 12

Please attempt all of the following problems before the due date. Your grade on this assignment will be calculated from the best three answers.

Problem 12.1

Consider the set $S^1$ of number pairs $\{(x, y) \mid x^2 + y^2 = 1\}$ and use the functions $X(x, y) = x$, and $Y(x, y) = y$ to define charts $(\varphi_i, U_i)$ on this set. Find all of the transition functions on the overlap regions and verify that the resulting structure on $S^1$ is a $C^\infty$ manifold.

Answer 12.1

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

Problem 12.2

Consider a curve described by coordinates

$$x(\theta) = \cos \theta, \quad y(\theta) = \sin \theta$$

and find a coordinate representation of the tangent vector to the curve at parameter value $\theta = 0$.

Answer 12.2

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

Problem 12.3

Consider a curve described by coordinates

$$x(\theta) = 1, \quad y(\theta) = \theta$$

and find a coordinate representation of the tangent vector to the curve at parameter value $\theta = 0$.

Answer 12.3
Problem 12.4

Suppose that a chart assigns the functions $x, y$ as coordinates on a manifold and describes a vector field by the components

\[
V^x (x, y) = -y \\
V^y (x, y) = x
\]

Find the integral curves of this vector field in the region away from the point $x = y = 0$.

Answer 12.4