**Hour Exam No.1**

Please attempt all of the following problems before the due date. All problems count the same even though some are more complex than others. Assume that $c = 1$ units are used throughout.

**Problem 1**

A photon with frequency $f$ in a particular "laboratory reference frame" is observed from a space probe that is moving at 20% of the speed of light in the laboratory frame. The photon is moving in the direction of the spacelike unit vector $\hat{n}$ while the probe is moving in the direction of the spacelike unit vector $\hat{b}$.

a. Find the four-momentum components of the photon in the laboratory frame.

**Answer 1a**

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

b. Find the components of the space probe’s four-velocity in the laboratory frame.

**Answer 1b**

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

C. Use results (a) and (b) to find an expression for the photon frequency that is observed from the space probe.

**Answer 1c**

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.
Problem 2

Show, using a \((+++\)) signature spacetime metric,

a. that the dot product of the four-momenta \(q, p\) of any two particles (massive or massless) must obey the inequality

\[ q \cdot p \leq 0. \]

Answer 2a

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

b. that a collection of massive particles cannot decay into a single photon.

Answer 2a

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.
Problem 3

Consider three linear mappings of a vector space $V$ into itself:

\[
A : V \rightarrow V \\
B : V \rightarrow V \\
C : V \rightarrow V
\]

The vector space is spanned by a set of $n$ basis vectors, $\{e_i\}$ and its dual space $\hat{V}$ is spanned by the dual basis forms $\{\omega^k\}$.

(a). Regard these mappings as tensors and write expressions for their tensor components in terms of the basis vectors and forms.

Answer 3a

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

(b). Regard the composite mapping $C \circ B \circ A$ as a tensor and find an expression for its components in terms of the components of $A, B, C$.

Answer 3b

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.
Problem 4

Suppose that an electrical current $I$ runs along the $z$ axis of a Minkowski coordinate system.

a. Express the resulting magnetic field components as functions of the Minkowski coordinates, $t, x, y, z$. It is OK to consult an E&M book here.

Answer 4a

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

b. Find the equation of motion of a positively charged particle that is moving through this magnetic field in the $x - z$ plane.

Answer 4b

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.
Problem 5

In spherical coordinates, the metric tensor on Minkowski spacetime has the form
\[ g = -dt \otimes dt + dr \otimes dr + r^2 \left( d\theta \otimes d\theta + \sin^2 \theta d\varphi \otimes d\varphi \right) \]

a. Write the components of this tensor and the inverse metric \( g^{-1} \) in the holonomic basis that corresponds to the coordinates
\[
\begin{align*}
x^0 &= t \\
x^1 &= r \\
x^2 &= \theta \\
x^3 &= \varphi
\end{align*}
\]

Answer 5a

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

b. Normalize the basis vectors \( \partial_t, \partial_r, \partial_\theta, \partial_\varphi \) and forms \( dt, d\theta, d\varphi \) to obtain an orthonormal (i.e. Minkowski) frame at each point. Write out the components of the transformation matrices that express the orthonormal basis vectors and forms in terms of the holonomic basis objects.

Answer 5b

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

C. The notes discuss the splitting of the Maxwell Field Tensor components into Electric and Magnetic field components in an orthonormal (Minkowski) frame. Show how this splitting works in a holonomic spherical coordinate frame. In other words, express \( f_{rs} \) in terms of \( E^k \) and \( B_j \) when all of these components are taken using the holonomic bases.
Answer 5b

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.
Problem 6

For one-forms $\alpha, \beta, \gamma$ and a vector $v$, consider the tensor

$$K = v \otimes (\alpha \otimes \beta \otimes \gamma + \beta \otimes \gamma \otimes \alpha + \gamma \otimes \alpha \otimes \beta)$$

a. Express the components of $\text{Tr}K$ in terms of the components of $\alpha, \beta, \gamma, v$.

Answer 6a

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

b. Express the tensor $\text{Tr}K$ as a combination of basis-independent objects such as dot products and tensor products of the objects $\alpha, \beta, \gamma, v$.

Answer 6b

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.