Exercise 9

Please attempt all of the following problems before the due date. Your grade on this assignment will be calculated from the best two answers.

Problem 9.1

Consider the set \( S^1 = \{ z \mid z^* z = 1 \} \) of unimodular complex numbers. Define the function \( \arg (z) \) by

\[
-\pi < \arg (z) < \pi \\
z = e^{i\arg(z)}
\]

and note that \( \arg (1) = 0 \) while \( \arg (-1) \) is not defined. Show that the functions defined by

\[
\varphi_1 (z) = \arg (z) ; z \neq -1 \\
\varphi_2 (z) = -\arg (-z) ; z \neq 1
\]

are a \( C^\infty \) atlas on the set \( S^1 \).

Answer 9.1

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

Problem 9.2

Consider a curve described by coordinates

\[
x (\theta) = \cos \theta, \quad y (\theta) = \sin (\theta)
\]

and find a coordinate representation of the tangent vector to the curve at parameter value \( \theta = 0 \).

Answer 9.2

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

Problem 9.3
Consider a curve described by coordinates

\[ x(\theta) = 1, \quad y(\theta) = \theta \]

and find a coordinate representation of the tangent vector to the curve at parameter value \( \theta = 0 \)

**Answer 9.3**

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.