• Suppose that you are given a network $G$, and you want to determine a path of shortest length that can start at either of the nodes $s_1$ or $s_2$ and can terminate at either of the nodes $t_1$ or $t_2$. How would you solve this problem?

Create network $G' = (N', A')$, where

$$
N' = N \cup \{s_0, t_0\}
$$

$$
A' = A \cup \{(s_0, s_1), (s_0, s_2), (t_1, t_0), (t_2, t_0)\}
$$

$$
c'_{ij} = \begin{cases} 
    c_{ij} & \text{if } (i, j) \in A \\
    0 & \text{otherwise.} 
\end{cases}
$$

Finding a shortest path from $s_0$ to $t_0$ solves the described problem; any $s_0$–$t_0$ path must go through one of $s_1$ and $s_2$, and through one of $t_1$ and $t_2$. Since arcs out of $s_0$ and into $t_0$ have zero length, they do not affect the length of the shortest path.

• Figure 4.15(b) in your text gives a road network in which all road segments are parallel to either the x-axis or the y-axis. The figure also gives the traversal costs of arcs. Suppose that we incur an additional cost (or penalty) of $a$ units every time we make a left turn. How would you find the lowest-cost route from node 1 to node 12 by solving a single-source shortest path problem?

Let $G = (N, A)$ be the original network given in Figure 4.15(b).
$G^* = (N^*, A^*)$, where

$N^* = \{i/j : (i,j) \in A\} \cup \{s_0,t_0\}$

$A^* = \{(i/j,j/k) : (i,j) \in A \text{ and } (j,k) \in A\}$

$= \{(s_0,1/2),(s_0,1/4),(9/12,t_0),(11/12,t_0)\}.$

Let

$$c^*_{i/j,j/k} = \begin{cases} c_{ij} & \text{if turning from arc } (i,j) \text{ onto arc } (j,k) \text{ in } G \\
& \text{is not a left turn and } k \neq 12 \\
& \text{if turning from arc } (i,j) \text{ onto arc } (j,k) \text{ in } G \\
& \text{is a left turn and } k \neq 12 \\
& \text{if turning from arc } (i,j) \text{ onto arc } (j,k) \text{ in } G \\
& \text{is not a left turn and } k = 12 \\
& \text{if turning from arc } (i,j) \text{ onto arc } (j,k) \text{ in } G \\
& \text{is a left turn and } k = 12 \\
& \end{cases}$$

and $c_{s_0,1/2} = c_{s_0,1/4} = c_{9/12,t_0} = c_{11/12,t_0} = 0$.

Notice that, in $G^*$, we consider the cost of left hand turns. We solve the single source shortest path problem with source $s_0$, and consider the shortest path from $s_0$ to $t_0$. A path

$$s_0 \rightarrow 1/i_1 \rightarrow i_1/i_2 \rightarrow \cdots \rightarrow i_{k-1}/i_k \rightarrow i_k/12 \rightarrow t_0$$

says to travel (in $G$) the arcs

$$(1,i_1), (i_1,i_2), \ldots, (i_k, 12).$$

The cost of this path in $G^*$ considers the cost of a left turn, if one is made. The cost of arc $(i/j,j/k)$ counts the cost of arc $(i,j)$, unless $k = 12$, in which case we include the cost of arc $(j,k)$ as well.

- **A. Suppose you wish to find a shortest walk from a source node $s$ to a sink node $t$ subject to the additional condition that the walk must visit a specified node $p$. How would you do this? Will this walk always be a path?**

Simply find the shortest path from $s$ to $p$ (call it $P_1$) and find the shortest path from $p$ to $t$ (call it $P_2$). Then the walk $P_1 - P_2$ gives the shortest path from $s$ to $t$ that visits $p$. This walk need not be a path (this can be seen by considering the graph $G = (N,A)$ with $N = \{1,2,3,4\}$ and $A = \{(1,2),(2,3),(2,4),(3,2)\}$, where $s = 1, t = 4, p = 3$ and all arc lengths are 1.)
B. Suppose you wish to find a shortest walk from a source node $s$ to a sink node $t$ subject to the additional condition that the walk must visit a specified arc $(p,q)$. How would you do this? Will this walk always be a path?

Find the shortest path from $s$ to $p$ (say, $P_1$) and the shortest path from $q$ to $t$ (say, $P_2$). The shortest walk from $s$ to $t$ using arc $(p,q)$ is $P_1-(p,q)-P_2$. This walk need not be a path (consider the graph given in part A with $s = 1, t = 4, p = 3, q = 2$).