1. Start with the first-order partial derivatives:

\[ f_x = -6xz \sin(x^2 - y), \quad f_y = 3z \sin(x^2 - y), \quad f_z = 3 \cos(x^2 - y) \]

Now proceed to the second partial derivatives:

\[ f_{xx} = -6z \sin(x^2 - y) - 12x^2 z \cos(x^2 - y) \]
\[ f_{xy} = f_{yx} = 6xz \cos(x^2 - y) \]
\[ f_{yy} = -3z \cos(x^2 - y) \]
\[ f_{yz} = f_{zy} = 3 \sin(x^2 - y) \]
\[ f_{zz} = 0. \]

2. The given surface is a level surface of the function \( F(z, y, z) = xy + yz + zx. \)
Take partial derivatives:

\[ F_x = y + z, \quad F_y = x + z, \quad F_z = y + x \]

The equation of the tangent plane is:

\[ F_x(1, 2, 1/3)(x - 1) + F_y(1, 2, 1/3)(y - 2) + F_z(1, 2, 1/3)(z - \frac{1}{3}) = 0 \]
\[ \frac{7}{3}(x - 1) + \frac{4}{3}(y - 2) + 3(z - \frac{1}{3}) = 0 \]
\[ 7x + 4y + 9z = 18. \]

3. Using the chain rule:

\[ \frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial u} = -y \sin(xy) - \cos y + 2u[-x \sin(xy) + x \sin y] \]
\[ \frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial v} = -(2v)[y \sin(xy) + \cos y] - x \sin(xy) + x \sin y. \]

4. The Lagrange system for this problem is:

\[ 4 = 2x\lambda, \quad -2 = 2y\lambda, \quad -2 = 2z\lambda, \quad x^2 + y^2 + z^2 = 6. \]
\[ x = \frac{2}{\lambda}, \quad y = \frac{-1}{\lambda}, \quad z = \frac{-1}{\lambda}, \quad \left(\frac{2}{\lambda}\right)^2 + \left(\frac{-1}{\lambda}\right)^2 + \left(\frac{-1}{\lambda}\right)^2 = 6. \]
The last equation gives \( \lambda = \pm 1 \). These values give two critical points that can be checked for max/min as follows:

\[
\begin{align*}
\lambda = 1 & \rightarrow (2, -1, -1) \rightarrow f(2, -1, -1) = 15 \quad \text{(Max)} \\
\lambda = -1 & \rightarrow (-2, 1, 1) \rightarrow f(-2, 1, 1) = -9 \quad \text{(Min)}.
\end{align*}
\]

5. The region of integration in the given integral is of Type II:

\[
D = \{(x, y) : 0 \leq y \leq 1, \ y^2 \leq x \leq 1\}
\]

This can be re-written as Type I (you may find it helpful to sketch \( D \)):

\[
D = \{(x, y) : 0 \leq x \leq 1, \ 0 \leq y \leq \sqrt{x}\}
\]

Now we calculate the integral in reverse order:

\[
\int_0^1 \int_{y^2}^1 y \cos(x^2) \, dx \, dy = \int_0^1 \int_0^{\sqrt{x}} y \cos(x^2) \, dy \, dx
\]

\[
= \int_0^1 \left[ \frac{y^2}{2} \right]_{y=0}^{y=\sqrt{x}} \cos(x^2) \, dx
\]

\[
= \frac{1}{2} \int_0^1 x \cos(x^2) \, dx \quad (u = x^2, \ du = 2x \, dx)
\]

\[
= \left[ \frac{1}{4} \sin(x^2) \right]_{x=0}^{x=1} = \frac{\sin 1}{4}.
\]

6. (a) The region of integration

\[
D = \{(x, y) : 0 \leq y \leq a, \ -\sqrt{a^2 - y^2} \leq x \leq \sqrt{a^2 - y^2}\}
\]

is the upper half of the disk \( x^2 + y^2 \leq a^2 \). In polar coordinates this is

\[
D = \{(r, \theta) : 0 \leq \theta \leq \pi, \ 0 \leq r \leq a\}
\]

So the integral becomes

\[
I = \int_0^\pi \int_0^a \frac{2}{\sqrt{4 + r^2}} \, r \, dr \, d\theta = \int_0^\pi d\theta \int_0^a r^{-1/2} \, du = 2\pi (\sqrt{4 + a^2} - 2)
\]
(b) Setting $I = \pi a$ gives the equation $2\pi(\sqrt{4+a^2} - 2) = \pi a$ which we can solve for $a$:

$$2(\sqrt{4+a^2} - 2) = a$$

$$\sqrt{4+a^2} = \frac{a}{2} + 2$$

$$4 + a^2 = \left(\frac{a}{2} + 2\right)^2 = \frac{a^2}{4} + 2a + 4$$

$$\frac{3a^2}{4} = 2a \rightarrow a = 0, \frac{8}{3}$$

Therefore, a positive solution is $a = \frac{8}{3}$.

7. The density in this problem is $\rho(x, y) = ky$ where $k$ is a positive constant. Hence

$$m = \int\int_D ky \, dA = \int_{-1}^{1} \int_{0}^{1-x^2} ky \, dy \, dx = \frac{k}{2} \int_{-1}^{1} (1-x^2)^2 \, dx = \frac{8k}{15}$$

where we used the expansion $(1-x^2)^2 = 1 - 2x^2 + x^4$ to do the last integral. Now we compute the coordinates of the center of mass. First, $\bar{x} = 0$ since the density $\rho = ky$ does not depend on the $x$-coordinate and also since the region $D$ under the parabola is symmetric with respect to the $y$-axis (drawing $D$ may be helpful to you). Next,

$$\bar{y} = \frac{1}{m} \int\int_D ky^2 \, dA = \frac{15}{8} \int_{-1}^{1} \int_{0}^{1-x^2} y^2 \, dy \, dx = \frac{15}{24} \int_{-1}^{1} (1-x^2)^3 \, dx = \frac{4}{7}$$

where we used the expansion $(1-x^2)^3 = 1 - 3x^2 + 3x^4 - x^6$ to do the last integral.

8. The volume over the rectangle $0 \leq x \leq 2$ and $0 \leq y \leq 1$ is given by the double integral

$$V = \int\int_D z \, dA = \int_{0}^{2} \int_{0}^{1} (6-x^2-2y^2) \, dy \, dx = \int_{0}^{2} \left(\frac{16}{3} - x^2\right) \, dx = 8.$$
9. The triple integral is calculated as follows:

\[
\iiint_E 12y \, dV = \int_0^1 \int_0^1 \int_0^{1+y^2} 12y \, dz \, dy \, dx \\
= \int_0^1 \int_0^1 12y(1 + x - y^2) \, dy \, dx \\
= \int_0^1 (3 + 6x) \, dx = 6.
\]

10. (a) \( \mathbf{F}(\mathbf{r}(t)) = \langle \cos(\pi t), -\sin(\pi t), -2t \rangle \) and \( \mathbf{r}'(t) = \langle \pi \cos(\pi t), -\pi \sin(\pi t), 1 \rangle \) so work is calculated as

\[
\int_C \mathbf{F} \cdot d\mathbf{r} = \int_0^1 \langle \cos(\pi t), -\sin(\pi t), -2t \rangle \cdot \langle \pi \cos(\pi t), -\pi \sin(\pi t), 1 \rangle \, dt \\
= \int_0^1 (\pi - 2t) \, dt = \pi - 1.
\]

(b) \( \mathbf{F}(\mathbf{r}(t)) = \langle t - 2, t - 1, -2t \rangle \) and \( \mathbf{r}'(t) = \langle -1, 1, 1 \rangle \) so work is

\[
\int_C \mathbf{F} \cdot d\mathbf{r} = \int_0^1 \langle t - 2, t - 1, -2t \rangle \cdot \langle -1, 1, 1 \rangle \, dt \\
= \int_0^1 (1 - 2t) \, dt = 0.
\]

11. (a) \( \mathbf{a}(t) = \mathbf{r}''(t) = \mathbf{i} + (t + 1)^{-2}\mathbf{j} \) so

\[
\mathbf{v}(t) = \mathbf{r}'(t) = \int [\mathbf{i} + (t + 1)^{-2}\mathbf{j}] \, dt = t\mathbf{i} - (t + 1)^{-1}\mathbf{j} + \mathbf{C}_1
\]

Since \( \mathbf{v}(0) = \mathbf{i} - \mathbf{j} + \mathbf{k} \) we calculate \( \mathbf{C}_1 = \mathbf{i} + \mathbf{k} \) and

\[
\mathbf{r}'(t) = (t + 1)\mathbf{i} - (t + 1)^{-1}\mathbf{j} + \mathbf{k}.
\]

Thus

\[
\mathbf{r}(t) = \int [(t + 1)\mathbf{i} - (t + 1)^{-1}\mathbf{j} + \mathbf{k}] \, dt = \frac{(t + 1)^2}{2}\mathbf{i} - \ln |t + 1|\mathbf{j} + tk + \mathbf{C}_2
\]

Using \( \mathbf{r}(0) = \mathbf{0} \), we calculate \( \mathbf{C}_2 = -(1/2)\mathbf{i} \). Therefore,

\[
\mathbf{r}(t) = \frac{t}{2}(t + 2)\mathbf{i} - \ln |t + 1|\mathbf{j} + tk.
\]
(b) At \( t = 1 \), velocity is \( \mathbf{v}(1) = 2\mathbf{i} - (1/2)\mathbf{j} + \mathbf{k} \), speed is \( |\mathbf{v}(1)| = \sqrt{21}/2 \) and the location of the particle is \( \mathbf{r}(1) = (3/2)\mathbf{i} - (\ln 2)\mathbf{j} + \mathbf{k} \). The curvature at \( t = 1 \) is determined as follows

\[
\kappa(1) = \frac{|\mathbf{r}'(1) \times \mathbf{r}''(1)|}{|\mathbf{r}'(1)|^3} = \frac{8 |(1/4)\mathbf{i} + \mathbf{j} + \mathbf{k}|}{21\sqrt{21}} = \frac{2\sqrt{33}}{21\sqrt{21}}.
\]

12. (a) The temperature gradient at the point \( (1, 2, 0) \) is computed as

\[
\nabla T = \langle 2e^{2x-y+z}, -e^{2x-y+z}, e^{2x-y+z} \rangle \to \nabla T(1, 2, 0) = \langle 2, -1, 1 \rangle
\]

The vector \( \mathbf{v} = 3\mathbf{i} - 4\mathbf{k} \) has length \( |\mathbf{v}| = 5 \) so the unit vector along \( \mathbf{v} \) is \( \mathbf{u} = \langle 3/5, 0, -4/5 \rangle \). Now the directional derivative, or the rate of change of temperature is

\[
D_\mathbf{u}T(1, 2, 0) = \langle 2, -1, 1 \rangle \cdot \left\langle \frac{3}{5}, 0, -\frac{4}{5} \right\rangle = \frac{2}{5} = 0.4.
\]

(b) The direction of maximum increase in temperature is the same as the direction of the gradient \( \nabla T(1, 2, 0) \). Therefore, the maximum possible rate of increase in temperature is \( |\nabla T(1, 2, 0)| = \sqrt{6} \approx 2.45 \).