Levene's Test for Equality of Variances

(Levene's Test for Homogeneity of Variances)

Levene's Test

A homogeneity-of-variance test that is less dependent on the assumption of normality than most tests. For each case, it computes the absolute difference between the value of that case and its cell mean and performs a one-way analysis of variance on those differences.

Assumptions: 1. The samples from the populations under consideration are independent.
2. The populations under consideration are approximately normally distributed.

Checking the assumptions

Assumption 1: Be sure that the samples were taken independently of one another.

Assumption 2: Construct side-by-side boxplots or Q-Q plots by treatment to assess normality.

Notation (One-Way ANOVA)

t = number of treatments [$t = k$ for one-way ANOVA; $t = kl$ for two-way ANOVA]

y_{ij} = sample observation j from treatment i ($j = 1, 2,\ldots, n_i$ and $i = 1, 2,\ldots, t$)

n_i = number of observations from treatment i (at least one n_i must be 3 or more)

$N = n_1 + n_2 + \cdots + n_t$ = total number of pieces of data (overall size of combined samples)

$\bar{y}_i$ = mean of sample data from treatment i

$D_{ij} = \left| y_{ij} - \bar{y}_i \right|$ = absolute deviation of observation j from treatment i mean

$\bar{D}_i$ = average of the n_i absolute deviations from treatment i

$\bar{D}$ = average of all N absolute deviations

Note: At least one of the treatments must have 3 or more observations or else the Levene's statistic will be undefined (the denominator will equal zero if each treatment has 1 or 2 observations).

Levene's Testing Procedure

Step 0: Check the assumptions.

Step 1: State the null and alternative hypotheses.

Null $H_0: \sigma_1^2 = \sigma_2^2 = \cdots = \sigma_t^2$

Alternative H_a: *Not all of the variances are equal.*

Step 2: Decide on the significance level, α.

$\alpha =$ ________

Step 3: Determine the critical value and rejection region.

	Classical Approach	P-value Approach
Critical Value:	$F_{\alpha,(df_1=t-1, df_2=N-t)}$	N/A
Rejection Region:	$F_{Levene} \geq F_{\alpha,(df_1=t-1, df_2=N-t)}$	p-value $\leq \alpha$

Note: $df_1 = t-1$ and $df_2 = N-t$

Step 4: Compute Levene's Statistic.

$$F_{Levene} = \frac{\dfrac{\sum_{i=1}^{t} n_i\left(\overline{D}_i - \overline{D}\right)^2}{(t-1)}}{\dfrac{\sum_{i=1}^{t}\sum_{j=1}^{n_i}\left(D_{ij} - \overline{D}_i\right)^2}{(N-t)}}$$

Note: For the computation of the p-value, $df_1 = t-1$ and $df_2 = N-t$.

Step 5: Make your decision.
If the value of the test statistic, F_{Levene}, falls in the rejection region or if p-value $\leq \alpha$, then reject H_0; otherwise, fail to reject H_0.

Step 6: State the conclusion in words.

Reject H_0: "At the $\alpha =$ ____ level of significance, there is enough evidence to conclude that (H_a in words)."

Fail to Reject H_0: "At the $\alpha =$ ____ level of significance, there is not enough evidence to conclude that (H_a in words)."