Analysis of Variance of Randomized Block Designs

Randomized Block Design (RBD)

Assumptions: 1. The samples from the populations under consideration are independent within each block.
2. The populations under consideration are normally distributed.
3. The standard deviations of the populations under consideration are equal; that is they are all the same value, σ.

Checking the assumptions

Assumption 1: Be sure that the samples were taken independently of one another.

Assumption 2: Construct side-by-side boxplots or Normal Q-Q plots by treatment to assess normality.

Assumption 3: Construct side-by-side boxplots by treatment or perform Levene's test for homogeneity of variances to assess equality of variances (equal spreads).

RBD ANOVA Identity

$$SSTo = SSTr + SSBl + SSE$$

One-Way ANOVA Sums of Squares

Sum of Squares	Defining Formula	Short-cut Formula
Total (*SSTo*)	$\sum_{i=1}^{k}\sum_{b=1}^{l}\left(y_{ib}-\bar{\bar{y}}\right)^2$	$\sum_{i=1}^{k}\sum_{b=1}^{l} y_{ib}^2 - \frac{(T)^2}{N}$
Treatments (*SSTr*)	$l\sum_{i=1}^{k}\left(\bar{y}_i-\bar{\bar{y}}\right)^2$	$\sum_{i=1}^{k}\frac{(T_i)^2}{l} - \frac{(T)^2}{N}$
Blocks (*SSBl*)	$k\sum_{b=1}^{l}\left(\bar{b}_b-\bar{\bar{y}}\right)^2$	$\sum_{b=1}^{l}\frac{(B_b)^2}{k} - \frac{(T)^2}{N}$
Error (*SSE*)	$SSE = SSTo - SSTr - SSBl$	$SSE = SSTo - SSTr - SSBl$

y_{ib} = sample observation from treatment i in block b ($b = 1, 2,\ldots, l$ and $i = 1, 2,\ldots, k$)

N = total number of pieces of data (overall size of combined samples) $[N = kl]$

T_i = total sum of sample data from treatment i

B_b = total sum of sample data from block b

$\bar{y}_i$ = mean of sample data from treatment i

$\bar{b}_b$ = mean of sample data from block b

T = total sum of all N pieces of data (overall sum / grand total)

Post Hoc Multiple Comparisons *(when ANOVA determines the means might be different)*

The Tukey-Kramer Studentized Range Method (Tukey's W)

Step 2: Find $q_{(k,df)}$, where k is the number of treatments, and $df = (k - 1)(l - 1)$. Use Table VIII on page 692 to obtain the value.

Step 3: Obtain the margin(s) of error of the confidence interval for $\mu_i - \mu_j$:

$$W = q_{(k,df_{Error})}\sqrt{\frac{MSE}{l}}$$

One-Way ANOVA Testing Procedure

Step 0: Check the assumptions.

Step 1: State the null and alternative hypotheses.

Null H_0: $\mu_1 = \mu_2 = \cdots = \mu_k$

Alternative H_a: *Not all means are equal.*

Step 2: Decide on the significance level, α.

α = ________

Step 3: Determine the critical value and rejection region.

	Classical Approach	*P-value Approach*
Critical Value:	$F_{\alpha,(df_1 = k-1, df_2 = (k-1)(l-1))}$	N/A
Rejection Region:	$F \geq F_{\alpha,(df_1 = k-1, df_2 = (k-1)(l-1))}$	p-value $\leq \alpha$

Note: $df_1 = k - 1$ and $df_2 = (k-1)(l-1)$

Step 4: Construct the one-way ANOVA table.

Source	*df*	*SS*	*MS* = *SS*/*df*	*F*-statistic	*p*-value
Treatments	$k - 1$	*SSTr*	$s_{Tr}^2 = MSTr = \frac{SSTr}{k-1}$	$F = \frac{s_{Tr}^2}{s_E^2} = \frac{MSTr}{MSE}$	$P(f \geq F)$
Blocks	$l - 1$	*SSBl*	$s_{Bl}^2 = MSBl = \frac{SSBl}{l-1}$		
Error	$(k-1)(l-1)$	*SSE*	$s_E^2 = MSE = \frac{SSE}{(k-1)(l-1)}$		
Total	$N - 1$	*SSTo*			

Note: For the computation of the p-value, $df_1 = k - 1$ and $df_2 = (k-1)(l-1)$.

Step 5: Make your decision.
If the value of the test statistic, F, falls in the rejection region or if p-value $\leq \alpha$, then reject H_0; otherwise, fail to reject H_0.

Step 6: State the conclusion in words.

Reject H_0: "At the α = ____ level of significance, there is enough evidence to conclude that (H_a in words)."

Fail to Reject H_0: "At the α = ____ level of significance, there is not enough evidence to conclude that (H_a in words)."