CMSC691 Optimization, VCU
Fall 2016, Assignment 4: Duality and Approximation Algorithms

Due: Friday, December 2, at 11:59 PM.
Submit via Blackboard under Assignments tab.

Total marks: 30 marks.

1 Exercises

1. (10 marks) What is the dual program for the following general LP?

$$\begin{array}{ll} \min & c^T x \\ \text{s.t.} & Gx \preceq h \\ & Ax = b. \end{array}$$

   For full marks, you must show the steps taken to derive the dual (i.e. don't just state the dual program).

2. (10 marks) As mentioned in class, LPs always satisfy strong duality, but SDPs do not. In this question, we will study an example where strong duality fails. Consider the following primal SDP:

$$\begin{array}{ll} \min & x_2 \\ \text{s.t.} & \begin{pmatrix} x_2+1 & 0 & 0 \\ 0 & x_1 & x_2 \\ 0 & x_2 & 0 \end{pmatrix} \succeq 0. \\ & x \in \mathbb{R}^n. \end{array}$$

   (a) (3 marks) Derive the primal optimal value for this SDP. (Hint: Use the fact that a block diagonal matrix is PSD iff all of its blocks are PSD. Then, apply the Schur complement to study the bottom right 2×2 block of the SDP constraint above, i.e. this will reveal that x_2 must take on a certain value.)

   (b) (3 marks) What is the dual SDP?

   (c) (3 marks) What is the dual optimal value?

   (d) (1 mark) Why does this example violate strong duality?

3. (10 marks) Give a strict quadratic program for the MAX-k-CUT problem defined below, and show how to relax it to an SDP. (You don't need to analyze the approximation ratio obtained by your SDP.)

   Definition 1 (MAX-k-CUT)**.** *Given an undirected graph $G = (V, E)$ with non-negative edge costs, and an integer k, find a partition of V into sets $S_1, \ldots, S_k$ so that the total cost of edges running between the sets is maximized.*

   In other words, MAX-k-CUT is analogous to MAX-CUT, except now we wish to partition the vertices into k sets, not 2.