CMSC 303 Introduction to Theory of Computation, VCU
Fall 2017, Assignment 5
Due: Thursday, November 2, 2017 in class

Total marks: 50 marks + 5 marks bonus for typing your solutions in LaTeX.

Unless otherwise noted, the alphabet for all questions below is assumed to be $\Sigma = \{0, 1\}$.

1. [5 marks] This question asks you to examine the formal definitions of a TM and related concepts closely. Based on these definitions, answer the following.
   (a) A *configuration* of a Turing Machine (TM) consists of three things. What are these three things?
   (b) Can input alphabet Σ contain the blank symbol $\sqcup$? Why or why not?
   (c) The tape is infinite. Is the tape alphabet infinite?
   (d) Can a Turing machine's head *ever* be in the same location in two successive steps?
   (e) What is the difference between a decidable language and a Turing-recognizable language?

2. [8 marks] This question gets you to practice describing TM's at a semi-low level. Let
$$L = \{0^n 1^n 2^n \mid n \geq 0\},$$
which is a non-context-free language (i.e. it has no PDA). Give an implementation-level description of a TM that decides L. By *implementation-level description*, we mean a description similar to Example 3.11 in the text (i.e. describe how the machine's head would move around, whether the head might mark certain tape cells, etc.... Please do *not* draw a full state diagram (for your sake and for ours)).

3. [10 marks] This question investigates variants of our standard TM model from class.
   (a) [5 marks] Consider a TM which, if it ever attempts to move the head left while on the first cell, self-destructs, i.e. a "self-destructing TM". Show that a self-destructing TM can simulate a standard TM.
   (b) [5 marks] Consider a TM whose tape is infinite in *both* directions (i.e. you can move left or right infinitely many spaces on the tape). We call this a TM with *doubly infinite tape*. Show that a standard TM can simulate a TM with doubly infinite tape.

4. [15 marks] This question studies closure properties of the decidable and Turing-recognizable languages.
   (a) [5 marks] Show that the set of decidable languages is closed under complement.
   (b) [5 marks] Show that the set of decidable languages is closed under concatenation.
   (c) [5 marks] Show that the set of Turing-recognizable languages is closed under concatenation.

5. [12 marks] This question allows you to explore variants of the computational models we've defined in class. Let a k-PDA be a pushdown automaton that has k stacks. In this sense, a 0-PDA is an NFA and a 1-PDA is a conventional PDA. We know that 1-PDAs are more powerful (recognize a larger class of languages) than 0-PDAs.
   (a) [6 marks] Show that 2-PDAs are more powerful than 1-PDAs. (Hint: Recall from A4 that the language $L = \{a^n b^n c^n \mid n \geq 0\}$ is not context-free.)
   (b) [6 marks] Show that 3-PDAs are not more powerful than 2-PDAs. (Hint: Show how to simulate 3 stacks using just 2 stacks.)