Directions This is just a quick quiz to test your knowledge of various definitions. Most questions are short-answer. You need not explain your work unless asked.

1. Short answer. Write each of the following sets by listing its elements between braces or denoting it with a familiar symbol or symbols.

   (a) \( \{ x \in \mathbb{Z} : |3x| \leq 10 \} = \{-3, -2, -1, 0, 1, 2, 3\} \)

   (b) \([5, 7] \cap [7, 10] = \{7\}\)

   (c) \(\{ x \in \mathbb{R} : \sin(\pi x) = 0\} - \mathbb{Z} = \mathbb{Z} - \mathbb{Z} = \emptyset\)

   (d) \(\mathcal{P}(\{1, 2\} \times \{\emptyset\}) = \mathcal{P}(\{1, \emptyset\}, (2, \emptyset)) = \{\emptyset, \{(1, \emptyset)\}, \{(2, \emptyset)\}, \{(1, \emptyset), (2, \emptyset)\}\}\).

   (e) \(\bigcap_{n \in \mathbb{N}} [3, 5 + 1/n] = [3, 5]\)

   (f) \((\{0, 3\} \times \mathbb{N}) \cap (\mathbb{N} \times \{5, 6\}) = \{(3, 5), (3, 6)\}\)

   (g) \((\mathbb{R} - \mathbb{N}) \cap \mathbb{Z} = \{0, -1, -2, -3, -4, -5, \ldots\}\)

   (h) \(\{X : X \subseteq \{3, 4\} \cap X\} = \{\emptyset, \{3\}, \{4\}, \{3, 4\}\}\)

2. Short answer. Write the following sets in set-builder notation.

   (a) \(\{\ldots, -3, 2, 7, 12, 17, 22, 27, \ldots\} = \{2 + 5n : n \in \mathbb{Z}\}\)

   (b) \(\left\{ \frac{1}{3}, \frac{2}{9}, \frac{3}{27}, \frac{4}{81}, \ldots \right\} = \left\{ \frac{n}{3^n} : n \in \mathbb{Z} \right\}\)
3. Write a truth table for the expression: \((P \iff Q) \Rightarrow \neg (P \lor Q)\)

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<th>P \lor Q</th>
<th>\neg (P \lor Q)</th>
<th>(P \iff Q) \Rightarrow \neg (P \lor Q)</th>
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4. Consider the following statement:
For every subset \(X \subseteq \mathbb{N}\), there is a subset \(Y \subseteq \mathbb{N}\) for which \(|X - Y| = 1\).

(a) Is this statement true or false? Explain.
This statement is FALSE. Consider that \(X\) could be the empty set. In that case \(|X - Y| = 0 \neq 1\).

(b) Write the statement in symbolic form.
\(\forall X \subseteq \mathbb{N}, \exists Y \subseteq \mathbb{N}, |X - Y| = 1\)

(c) Write the negation of the statement as an English sentence.
The negation is:
\(\sim (\forall X \subseteq \mathbb{N}, \exists Y \subseteq \mathbb{N}, |X - Y|) = \exists X \subseteq \mathbb{N}, \sim (\exists Y \subseteq \mathbb{N}, |X - Y|) = \exists X \subseteq \mathbb{N}, \forall Y \subseteq \mathbb{N}, \sim (|X - Y|) = \exists X \subseteq \mathbb{N}, \forall Y \subseteq \mathbb{N}, |X - Y| \neq 1\)

Final Answer: There is a subset \(X \subseteq \mathbb{N}\) for which \(|X - Y| \neq 1\) for every subset \(Y \subseteq \mathbb{N}\).

5. This question involves lists made from the symbols \(A,B,C,D,E,F\). How many length-6 lists can be made from these symbols if repetition is allowed and the first or last entry must be an \(A\)? (Show your work. It is OK to leave your final answer in unsimplified form.)

**METHOD 1:**
(All Lists) − (Those Lists where first and last entry is not \(A\)) = \(6^6 - 5 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 5 = 6^6 - 5^2 \cdot 6^4\)
\(= 6^4(6^2 - 5^2) = 11 \cdot 6^4\).

**METHOD 2:**
Lists will be of three types:
Type 1: (A, anything, anything, anything, anything, A) ......................... \(6^4\) of these
Type 2: (A, anything, anything, anything, anything, not A) ......................... \(6^4 \cdot 5\) of these
Type 3: (not A, anything, anything, anything, anything, A) ......................... \(5 \cdot 6^4\) of these

The total number of lists is thus \(6^4 + 10 \cdot 6^4 = 11 \cdot 6^4\)