Test Two | Advanced Linear Algebra | November 1, 2018
MATH 610
Name: ______________________________ | R. Hammack | Score: ______

Directions: All spaces are finite dimensional. Each question (including parts) is 10 points.

1. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$.

   (a) Find the characteristic polynomial χ_A of A. (Show your work.)

   (b) Find the minimum polynomial m_A of A. (Explain your ressoning.)

   (c) Is A diagonalizable? (Explain.)

2. Give an example of an operator $T : \mathbb{R}^2 \to \mathbb{R}^2$ for which $\mathbb{R}^2 = \text{Range}(T) \oplus \text{Null}(T)$ but T is not a projection.

3. Find the matrix (relative to the standard basis) for the projection that projects $\mathbb{R}^2$ to Span $\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)$ along Span $\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$.

4. Suppose $V = W_1 \oplus W_2$. Let E_1 be a projection to W_1. Let E_2 be a projection to W_2.

(a) Is it necessarily true that $E_1 E_2 = O$? Explain.

(b) Is it necessarily true that $E_1 + E_2$ is a projection? Explain.

5. Suppose $E, T \in L(V, V)$ and E is a projection onto the subspace $W \subseteq V$.
   Prove that W is T-invariant if and only if $ETE = TE$.

6. Suppose $T \in L(V, V)$ and every subspace of V is T-invariant. Prove that T is a scalar multiple of the identity.

7. Let V be the vector space of $n \times n$ matrices over a field $\mathbb{F}$ and let $A \in V$ be a fixed matrix.
Define a linear operator $T\colon V \to V$ as $T(X) = AX$.
Show that the minimum polynomial of T equals the minimum polynomial of A.