1. Draw the subgroup lattice for $\mathbb{Z}_{18}$.

2. List the elements of the cyclic subgroup $\langle -i \rangle$ of $\mathbb{C}^*$.

3. Find the order of the largest cyclic subgroup of the symmetric group $S_{10}$.
   Consider the element $(1,2,3,4,5)(6,7,8)(9,10)$. 
4. Consider the set \( H = \{ \sigma \in S_5 \mid \sigma(3) = 3 \} \).

(a) \(|H| =

(b) Explain why \( H \) is a subgroup of \( S_5 \).

(c) Is \( H \) a normal subgroup of \( S_5 \)? Explain.

(d) How many left cosets of \( H \) are there in \( S_5 \)?

5. List all the nonisomorphic groups of order 180.

6. Find the order of \((3,6,9)\) in \( \mathbb{Z}_4 \times \mathbb{Z}_{12} \times \mathbb{Z}_{15} \).
7. Are the groups $\mathbb{Z}_8 \times \mathbb{Z}_{10} \times \mathbb{Z}_3$ and $\mathbb{Z}_8 \times \mathbb{Z}_2 \times \mathbb{Z}_{15}$ isomorphic? Why or why not?

8. Find the kernel of the homomorphism $\phi: \mathbb{Z} \rightarrow \mathbb{Z}_8$ for which $\phi(1)=6$.

9. Find the kernel of the homomorphism $\phi: \mathbb{Z}_{40} \rightarrow \mathbb{Z}_5 \times \mathbb{Z}_8$ for which $\phi(1)=(1,4)$.

10. (a) List the units in the ring $\mathbb{Z}_{12}$.
    (b) List the zero divisors in the ring $\mathbb{Z}_{12}$.
    (c) List the prime ideals in the ring $\mathbb{Z}_{12}$. 
11. What familiar group is \((Z_4 \times Z_6)/\langle (2, 3) \rangle\) isomorphic to?

12. Explain why \(C^*/U \cong \mathbb{R}^+\).

13. Is \(2x^3 + x^2 + 2x + 2\) an irreducible polynomial in \(Z_5[x]\)? If not, write it as a product of irreducible polynomials.
14. Find all $c \in \mathbb{Z}_3$ for which $\mathbb{Z}_3[x]/\langle x^2 + c \rangle$ is a field.

15. Prove that if $G$ is a finite group with identity $e$, and $m = |G|$, then $x^m = e$ for any element $x \in G$.

16. Suppose that $G$ is a group with identity $e$. Prove that if $x^2 = e$ for every element $x$ in $G$, then $G$ is abelian.
17. Prove that if $G$ is an abelian group, then the set of all elements $x \in G$ for which $x^2 = e$ form a subgroup of $G$.

18. Prove that the units of a ring with unity form a multiplicative group.