1. Suppose $A$ is a fixed $2 \times 2$ matrix. Show that the set $W = \{ X : AX =XA \}$ is a subspace of $M_{2,2}$.

   (a) Suppose that $B$ and $C$ are matrices in the set $W$.
   This means $AB = BA$ and $AC = CA$.
   Then $A(B + C) = AB + AC = BA + CA = (B + C)A$.
   And $A(B + C) = (B + C)A$ means that $B + C \in W$.
   Therefore $W$ is closed under addition.

   (b) Suppose that $B \in W$ and $c \in \mathbb{R}$.
   The fact that $B \in W$ means $AB = BA$.
   Observe that $A(cB) = c(AB) = c(BA) = (cB)A$.
   And $A(cB) = (cB)A$ means $cB \in W$.
   Therefore $W$ is closed under scalar multiplication.

Parts (a) and (b) above show that $W$ is closed under addition and scalar multiplication, so $W$ is a subspace.