☞ Quiz: Sections 2.1 & 2.2 | Linear Algebra MATH 310 R. Hammack | September 18, 2018

Name: ____________________ Score: 10

Directions: Please answer in the space provided. No calculators. Please put all phones, etc., away.

1. For this problem, $A = \begin{bmatrix} 3 & 1 & -5 \\ 4 & 2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$, and $D = \begin{bmatrix} 4 & -1 & -1 \end{bmatrix}$.

   Preform the indicated operations or state that they are not possible.

   (a) $AD^T = \begin{bmatrix} 3 & 1 & -5 \\ 4 & 2 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \\ -1 \end{bmatrix} = \boxed{\begin{bmatrix} 16 \\ 12 \end{bmatrix}}$

   (b) $AD^T - 2C = \begin{bmatrix} 16 \\ 12 \end{bmatrix} - 2\begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} 16 \\ 12 \end{bmatrix} - \begin{bmatrix} -4 \\ 8 \end{bmatrix} = \boxed{\begin{bmatrix} 20 \\ 4 \end{bmatrix}}$

   (c) $B^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}}$

   (d) $B^2 - 2B + I_2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}$

2. Suppose $\begin{bmatrix} w & x \\ y & z \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix}$. Find $\begin{bmatrix} w & x \\ y & z \end{bmatrix}$.

   Solution: Multiplying the matrices on the left gives $\begin{bmatrix} w+3x & 2w+4x \\ y+3z & 2y+4z \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix}$. This yields the system

$$\left\{\begin{array}{rcrcrcrcl} w & + & 3x & & & & & = & 1 \\ 2w & + & 4x & & & & & = & 4 \\ & & & & y & + & 3z & = & 0 \\ & & & & 2y & + & 4z & = & 2 \end{array}\right.$$

   We need to solve this to find w, x, y and z. Using our usual method,

$$\begin{bmatrix} 1 & 3 & 0 & 0 & 1 \\ 2 & 4 & 0 & 0 & 4 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 2 & 4 & 2 \end{bmatrix} \xrightarrow[R_4 - 2R_3 \to R_4]{R_2 - 2R_1 \to R_2} \begin{bmatrix} 1 & 3 & 0 & 0 & 1 \\ 0 & -2 & 0 & 0 & 2 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & -2 & 2 \end{bmatrix} \xrightarrow[-\frac{1}{2}R_4 \to R_4]{-\frac{1}{2}R_2 \to R_2} \begin{bmatrix} 1 & 3 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \xrightarrow[R_3 - 3R_4 \to R_3]{R_1 - 3R_2 \to R_1}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \Longrightarrow \left\{\begin{array}{rcr} w & = & 4 \\ x & = & -1 \\ y & = & 3 \\ z & = & -1 \end{array}\right.$$

   Answer: $\begin{bmatrix} w & x \\ y & z \end{bmatrix} = \boxed{\begin{bmatrix} 4 & -1 \\ 3 & -1 \end{bmatrix}}$