

Name: RichardScore: 10

Directions: Please answer in the space provided. Please show all of your work and indicate your row operations. No calculators. Please put all phones, etc., away.

1. Use either *Gaussian Elimination* or *Gauss-Jordan Elimination* to solve the following system.

$$\begin{cases} 2w + 4x + y + 8z = 4 \\ w + 2x + \quad + 2z = 1 \\ w + 2x + y + 6z = 3 \end{cases}$$

$$\left[\begin{array}{cccc|c} 2 & 4 & 1 & 8 & 4 \\ 1 & 2 & 0 & 2 & 1 \\ 1 & 2 & 1 & 6 & 3 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cccc|c} 1 & 2 & 0 & 2 & 1 \\ 2 & 4 & 1 & 8 & 4 \\ 1 & 2 & 1 & 6 & 3 \end{array} \right] \xrightarrow{\begin{array}{l} R_2 - 2R_1 \rightarrow R_2 \\ R_3 - R_1 \rightarrow R_3 \end{array}}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 2 & 1 \\ 0 & 0 & 1 & 4 & 2 \\ 0 & 0 & 1 & 4 & 2 \end{array} \right] \xrightarrow{\begin{array}{l} R_3 - R_2 \rightarrow R_3 \\ R_2 \leftrightarrow R_3 \end{array}} \left[\begin{array}{cccc|c} 1 & 2 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 & 2 \end{array} \right]$$

↓

$$\begin{cases} w + 2x + 2z = 1 \\ y + 4z = 2 \end{cases}$$

$$\begin{cases} w = 1 - 2x - 2z \\ x = \text{"free"} \\ y = 2 - 4z \\ z = \text{"free"} \end{cases}$$

Let $x = s$ and $z = t$.

Solutions $(w, x, y, z) = (1 - 2s - 2t, s, 2 - 4t, t)$
where $s, t \in \mathbb{R}$