Midterm | Linear Algebra | October 16, 2018
MATH 310
Name: ______________ R. Hammack Score: ______

Answer in the space provided. No calculators. Please put all phones, etc., away. Each problem is 10 points.

1. For this problem, $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ 2 & -2 \end{bmatrix}$, $C = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, and $D = \begin{bmatrix} 3 & 1 \end{bmatrix}$.

   Preform the indicated operations or state that they are not possible.

   (a) $A^T C =$

   (b) $DB^2 =$

   (c) $B^{-1} =$

   (d) $|B| =$

   (e) $\left|B^5\right| =$

2. Suppose X and Y are matrices for which the product XY is defined, and $c \in \mathbb{R}$.

   (a) If $XY = O$, is necessarily true that $X = O$ or $Y = O$? Justify your answer.

   (b) If $cY = O$, is necessarily true that $c = 0$ or $Y = O$?

3. Solve the system $\begin{cases} 2w - x + 8y - 4z = 4 \\ w + x + 4y - 2z = 5 \\ w - 2x + 4y - 2z = -1 \end{cases}$

4. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 3 \\ 3 & 3 & 4 \end{bmatrix}$.

5. Suppose that $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 3 \\ 3 & 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$. Use your answer from problem 4 above to find $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

6. Consider the two vectors $\mathbf{u}_1 = (2, 1, 2)$ and $\mathbf{u}_2 = (4, -1, 2)$ in $\mathbb{R}^3$.
Is the vector $\mathbf{v} = (2, 7, 6)$ a linear combination of $\mathbf{u}_1$ and $\mathbf{u}_2$? Explain.

7. Find the value(s) of x for which the matrix $\begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & 0 \\ 4 & 2 & x \end{bmatrix}$ is not invertible.

8. Find the 2×2 matrix A for which $A \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.

9. An $n \times n$ (square) matrix A is said to be **skew-symmetric** if $A^T = -A$.
Suppose n is odd and A is skew-symmetric. Find $\det(A)$.

10. In this problem we regard $\mathbb{R}^n$ as a set of column vectors, that is, $\mathbb{R}^n = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \ : \ x_1, x_2, \ldots, x_n \in \mathbb{R} \right\}$.

Let A be fixed $n \times n$ matrix. Show that the set $W = \{\mathbf{x} \in \mathbb{R}^n \ : \ A\mathbf{x} = -\mathbf{x}\}$ is a subspace of $\mathbb{R}^n$.