Name: ________________________________

Score: __________

Directions. Solve the following questions in the space provided. Unless noted otherwise, you must show your work to receive full credit. This is a closed-book, closed-notes test. Calculators, computers, etc., are not used. Put a your final answer in a box, where appropriate.

1. (24 points) Let \( \mathbf{u} = \langle 2, -2, 3 \rangle \) and \( \mathbf{v} = \langle 0, -2, 1 \rangle \).

   (a) \( \mathbf{u} \cdot \mathbf{v} = \)

   (b) \( \mathbf{u} \times \mathbf{v} = \)

   (c) \( |\mathbf{u}| = \)

   (d) \( |\mathbf{v}| = \)

   (e) Find \( \cos \theta \), where \( \theta \) is the angle between \( \mathbf{u} \) and \( \mathbf{v} \).

   (f) Find \( x \), where \( 2x - \mathbf{v} = 3\mathbf{u} \).

6. (10 pts.) Suppose \( f(x, y) = \frac{\sqrt{x - y}}{1 - x^2 - y^2} \).
   Sketch the domain of this function.
2. (10 pts.) Find the equation for the plane containing the point \((1, 4, 2)\) and the line \(r(t) = (1 - 2t)i + (2 + t)j + (5 - t)k\).

3. (16 pts.) Consider the triangle in space whose vertices are the points \(A(1, 1, 4), B(-1, 3, 3)\) and \(C(3, 2, 1)\).

   (a) Find a vector normal to the plane that the triangle lies in.

   (b) Find the area of the triangle \(ABC\).
4. (30 pts.)

(a) Find a (non-zero) vector orthogonal to \( \mathbf{v} = \langle 5, 4, -7 \rangle \).

(b) \[ \int_{\pi/4}^{\pi} \langle \sin t, 1, \sin t \cos t \rangle \, dt = \]

(c) Compute the arc length of the helix \( \mathbf{r}(t) = \langle t, \sin t, \cos t \rangle \) between \( t = 0 \) and \( t = 4\pi \).

5. (10 pts.) An object moving in space has acceleration \( \mathbf{a}(t) = \langle 1, \frac{t}{2}, 1 \rangle \) feet per second per second at time \( t \) seconds. Suppose that at time \( t = 0 \) it is at the origin and has velocity vector \( \langle 1, 1, 2 \rangle \). Find the velocity function \( \mathbf{v}(t) \) and its position function \( \mathbf{r}(t) \).