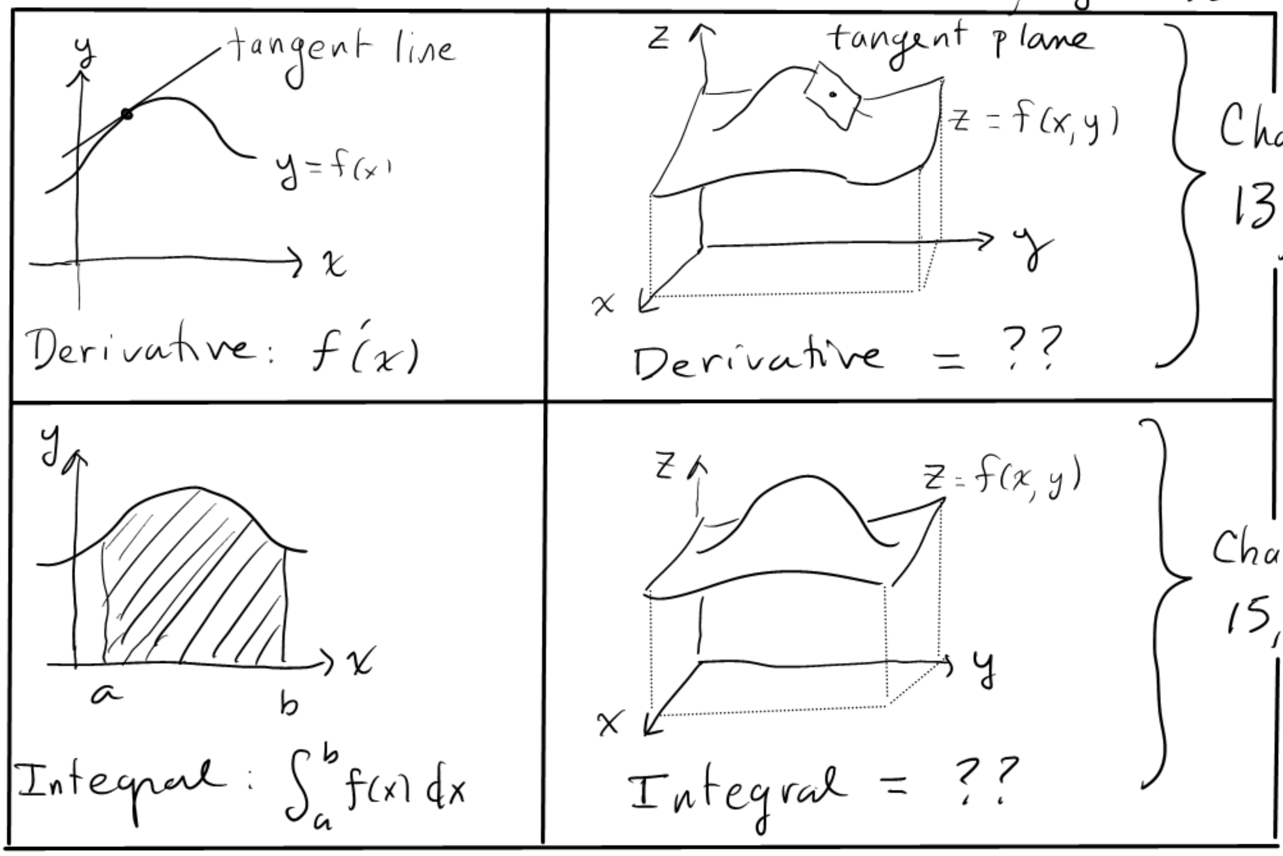
MATH 307 DAY 1

The goal of MATH 307 is to generalize the ideas of differentiation and integration (MATH 200, 201) to functions with more than one variable.

One variable: $f(x)$ More than one variable, e.g. $f(x,y)$



Chapters 13, 14

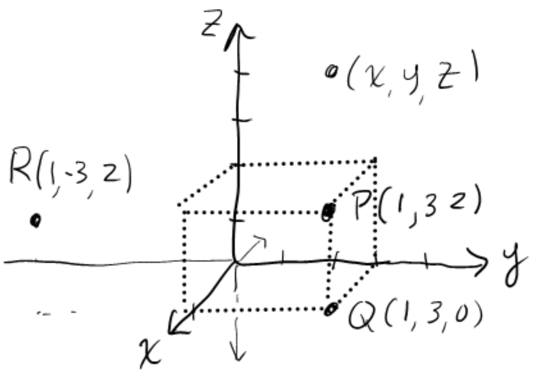
Chapters 15, 16

To make progress in these directions, we need a method of dealing with space. That's the purpose of our beginning chapter.

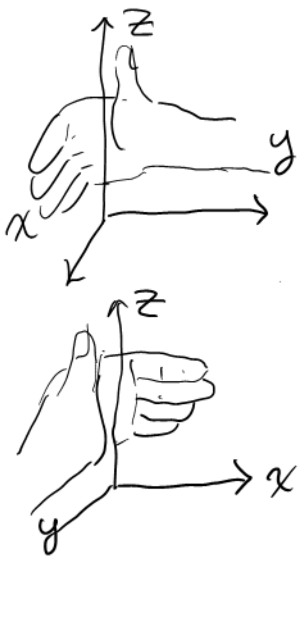
Chapter 12 Vectors and The Geometry of Space

Section 12.1 Three Dimensional Coordinates.

3-D space can be described with the xyz-coordinate system

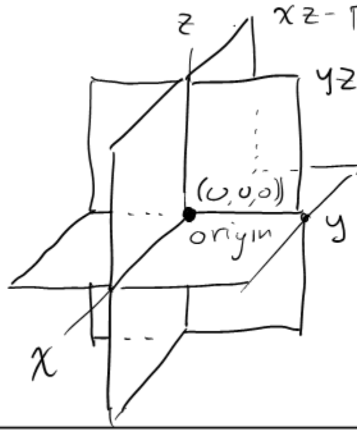


$$\begin{pmatrix}\text{Points in}\\ \text{space}\end{pmatrix} \Leftrightarrow \begin{pmatrix}\text{coordinates}\\ (x, y, z)\end{pmatrix}$$



Right hand system (we use this)

Left hand system (not this)



xz-plane ($y=0$; x,z vary)

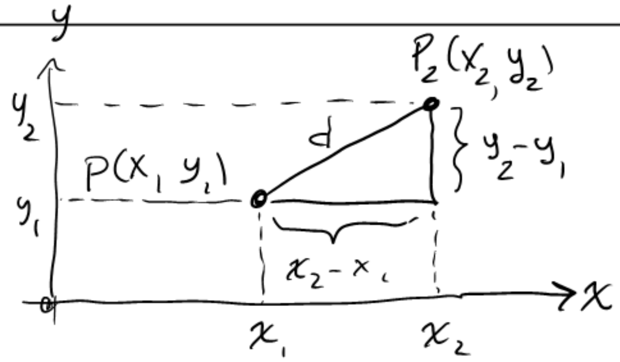
yz-plane ($x=0$; y,z vary)

xy-plane ($z=0$; x,y vary)

3 coordinate planes cut space into 8 octants.

2-D Pythagorean Theorem

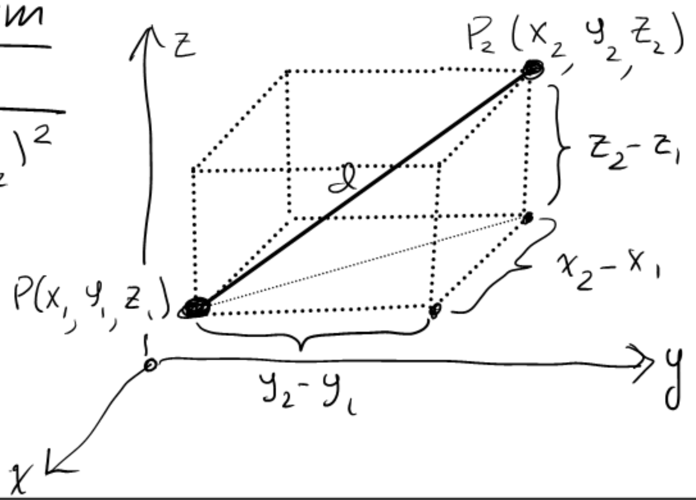
$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$



3-D Pythagorean Theorem

$$d = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$$

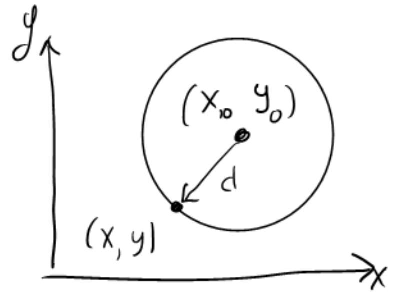
(Obtained by applying 2-D pythagorean theorem twice to triangles in indicated box. See text.)



From this we get the following:

Equation for circle of radius d centered at (x_0, y_0):

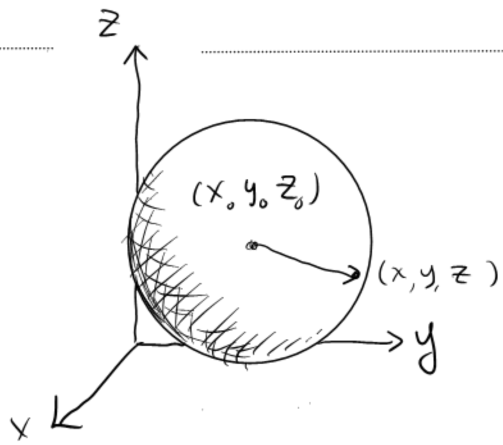
$$(x-x_0)^2 + (y-y_0)^2 = d^2$$



Equation for sphere of radius d centered at (x_0, y_0, z_0):

$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = d^2$$

(i.e. sphere is the set of all points (x, y, z) that satisfy this.)

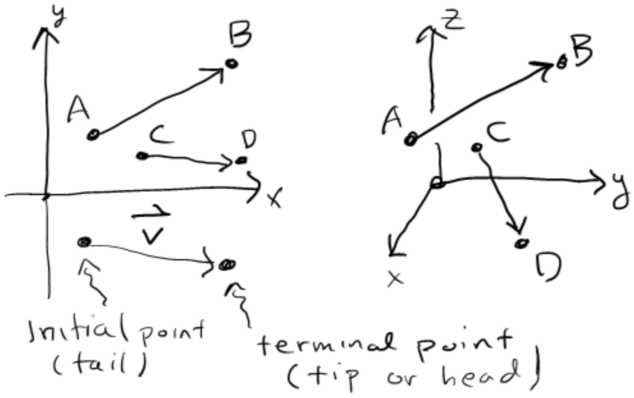


Read Examples in Text!

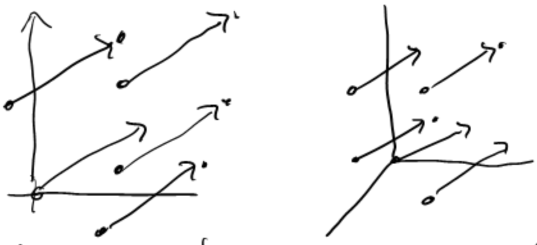
Section 12.2 Vectors

A **vector** in 2-D or 3-D is a directed line segment $\overrightarrow{AB}$ that has direction and length.

A vector $\mathbf{v} = \overrightarrow{AB}$ is often indicated by a boldface letter $\mathbf{v}$, $\mathbf{u}$, etc., or as $\vec{v}$, $\vec{u}$, etc. in handwriting.



Two vectors are considered **equal** if they have the same direction and length. Thus you can move a vector around — as long as it has the same direction and length it's considered the same vector.

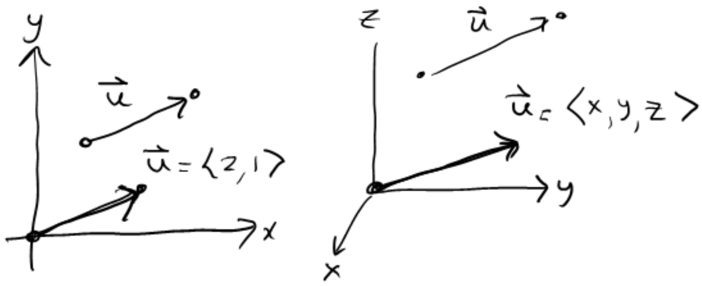


Equal vectors in 2-D

Equal vectors in 3-D.

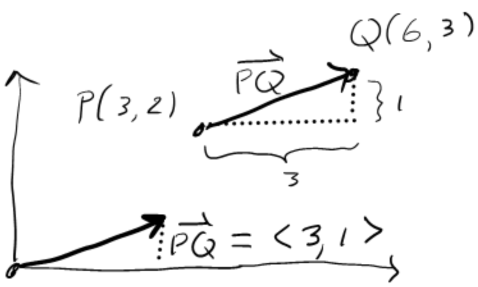
A vector whose tail is at the origin is said to be in **standard position**. Such a vector is completely specified by its terminal point (x, y) or (x, y, z). So its written as

$\vec{u} = \langle x, y\rangle$ or $\vec{u} = \langle x, y, z\rangle$.



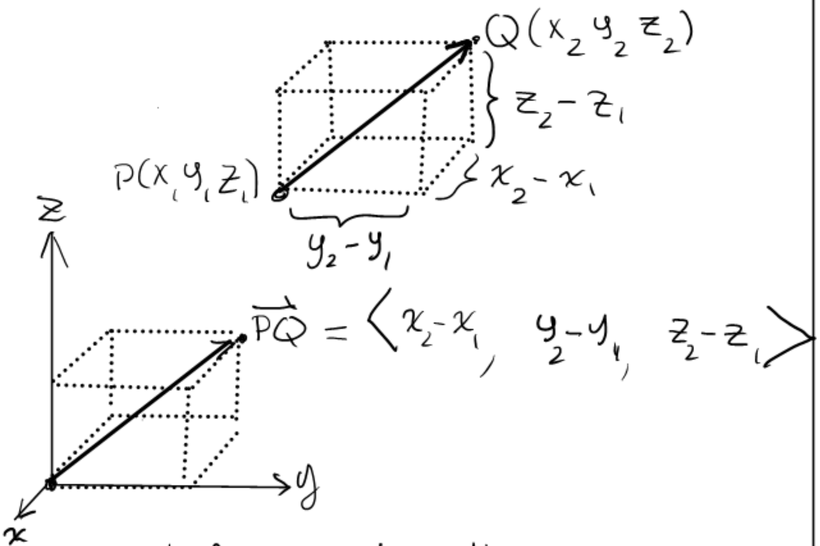
Vectors in standard position

Examples



Magnitude or length is

$$|\overrightarrow{PQ}| = \sqrt{3^2 + 1^2} = \sqrt{10}$$



$$\overrightarrow{PQ} = \langle x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1\rangle$$

Magnitude or length or norm is

$$|\overrightarrow{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

In general, if $\vec{v} = \langle v_1, v_2 \rangle$ then $|\vec{v}| = \sqrt{v_1^2 + v_2^2}$

if $\vec{v} = \langle v_1, v_2, v_3 \rangle$ then $|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

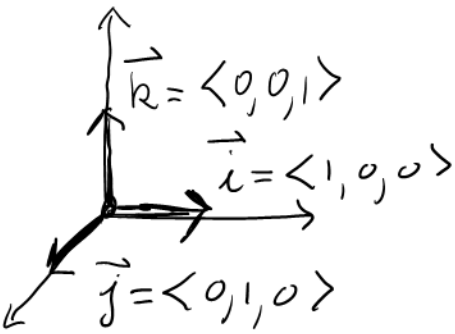
A <u>unit vector</u> is one that has length 1.

<u>Example</u> $\vec{v} = \left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$

$$|\vec{v}| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2} = \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = \sqrt{1} = 1.$$

thus $\vec{v}$ is a <u>unit vector</u>

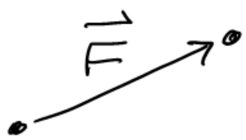
Here are three special unit vectors that come up a lot:



<u>Summary</u>

Vectors are significant because they are part of a mathematical language for describing and analyzing <u>space</u>

Furthermore, many physical phenomina are naturally modeled by vectors:



- <u>Force</u> is a vector
  It acts in a particular <u>direction</u> with a particular <u>magnitude</u>

- <u>Velocity</u> is a vector
  It's specified by <u>direction</u> (of motion) and magnitude (speed)

There are <u>many</u> more such examples

Next time we will continue with section 12.2.
Read this section !!