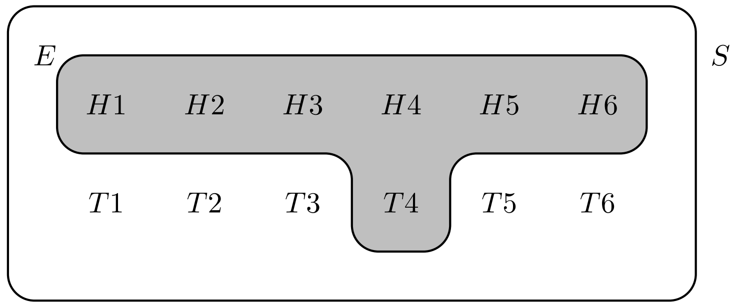
MATH 211 Test #2 ♠ October 31, 2019

Name:________________ R. Hammack Score:_______

Directions You must show your work to get full credit. This test is closed-book and closed-notes. No calculators or other electronic devices are allowed. Simplify your answers if it is easy to do so, but you may leave complex answers unsimplified. All you will need is something to write with.

1. (10 points) Toss a coin and then roll a 6-sided dice. Write out the sample space S for this experiment. Consider the event E : *The coin is heads or the dice is* ⚃ *(four).*
   Circle E in S. Find $p(E)$.



$$p(E) = \frac{|E|}{|S|} = \frac{7}{12} = \frac{3}{4} = \boxed{58.\overline{3}\%}$$

2. (10 points) Toss a dice 10 times in a row.
   What are the chances that exactly five of the tosses are ⚃ (four)?

The sample space S is the set of all length-10 lists made from the symbols ⚀,⚁,⚂,⚃,⚄,⚅, with repetition allowed. Thus $|S| = 6^{10}$.

Consider the event $E \subseteq S$ of exactly five ⚃'s.

To make a list in E, start with 10 empty spots.

Then pick five out of the ten spots for ⚃'s. You can do this in $\binom{10}{5}$ ways.

	⚃		⚃		⚃	⚃			⚃

Now there are 5 choices for each of the remaining five spots (anything but ⚃).

5	⚃	5	⚃	5	⚃	⚃	5	5	⚃

By the multiplication principle, $|E| = 5^5\binom{10}{5}$.

Thus $p(E) = \frac{|E|}{|S|} = \frac{5^5\binom{10}{5}}{6^{10}} = \frac{787500}{60466176} \approx \boxed{1.3\%}$

3. (10 points) The top card and the bottom card of a shuffled 52-card deck are removed.
You win a dollar if the top card a diamond or the bottom card is a heart.
What are your chances of winning?

Consider the following events:
A : The top card is a diamond
B : The bottom card is a heart

The answer will be $p(A \cup B)$.

$$\begin{aligned} p(A \cup B) &= p(A) + p(B) - p(A \cap B) \\ &= p(A) + p(B) - p(A) \cdot p(B|A) \\ &= \frac{13}{52} + \frac{13}{52} - \frac{13}{52} \cdot \frac{13}{51} \\ &\approx \mathbf{43.627}\% \end{aligned}$$

4. (10 points) A 5-card hand is dealt off a shuffled standard 52-card deck.
What is the probability that not all of the cards are red?

The sample space S is the set of all 5-element subsets of the deck of 52 cards, so $|S| = \binom{52}{5}$.

Let E be the event of all five cards being red, so $|E| = \binom{26}{5}$.
(The number of ways to choose 7 out of 26 red cards.)

Now, $\overline{E}$ is the event of not all the five cards being red. Thus our answer is

$$p(\overline{E}) = 1 - p(E) = 1 - \frac{|E|}{|S|} = 1 - \frac{\binom{26}{5}}{\binom{52}{5}} = 1 - \frac{\frac{P(26,5)}{5!}}{\frac{P(26,5)}{5!}} = 1 - \frac{P(26,5)}{P(52,7)} = 1 - \frac{506}{2499} \approx 79.75\%$$

5. (10 points) A box contains 8 red balls and 5 green balls. You reach in and remove two balls, one after the other. What is the probability that the two balls have the same color?

The sample space is $S = \{\, RR,\ RG,\ GR,\ GG \,\}$.
Thus the event of both balls having the same color is $E = \{RR, GG\}$.

Let A be the event of the first draw being R.
Let B be the event of the second draw being R.

So $\overline{A}$ be the event of the first draw being G.
So $\overline{B}$ be the event of the second draw being G.

$P(RR) = p(A \cap B) = p(A) \cdot p(B|A) = \frac{8}{13} \cdot \frac{7}{12}$.
$P(GG) = p(\overline{A} \cap \overline{B}) = p(\overline{A}) \cdot p(\overline{A}|\overline{B}) = \frac{5}{13} \cdot \frac{4}{14}$.

Then $p(E) = p\big(\{RR, GG\}\big) = p(RR) + p(GG) = \dfrac{8}{13} \cdot \dfrac{7}{12} + \dfrac{5}{13} \cdot \dfrac{4}{12} = \dfrac{8 \cdot 7 + 5 \cdot 4}{13 \cdot 12} = \dfrac{76}{156} \approx \boxed{\mathbf{48.71\%}}$

6. (10 points) Suppose $A, B \subseteq S$ are two events in the sample space S of some experiment.
Suppose $p(A) = 40\%,\ \ p(A|B) = 40\%$ and $p(B|A) = 50\%$.

(a) $p\left(\overline{A}\right) = 1 - p(A) = 1 - 0.4 = 0.6 = \boxed{\mathbf{60\%}}$

(b) Are A and B independent or dependent?
Because $p(A) = 40\%$ and $p(A|B) = 40\%$, $\boxed{A \text{ and } B \text{ are independent}}$.

(c) $p(A \cap B) = p(A) \cdot p(B|A) = (0.4)(0.5) = 0.2 = \boxed{\mathbf{20\%}}$

(d) $p(B) =$
Because $p(A \cap B) = p(B) \cdot p(A|B)$, the above information yields
$0.2 = p(B) \cdot 0.4$, so $p(B) = \dfrac{0.2}{0.4} = 0.5 = \boxed{\mathbf{50\%}}$.

(e) $p(A \cup B) = p(A) + p(B) - p(A \cap B) = 0.4 + 0.5 - 0.2 = \boxed{\mathbf{70\%}}$.

7. (10 points) A coin is tossed three times in a row, and there more heads than tails. What is the probability that the first toss is a tail?

The sample space can be considered to be $S = \{HHH, HHT, HTH, THH\}$.
The event of the first toss being a tail is $E = \{THH\} \subseteq S$.

Thus $p(E) = \dfrac{|E|}{|S|} = \dfrac{1}{4} = \boxed{\mathbf{25\%}}$.

8. (10 points) Give the output for the following chunk of pseudocode.

```
y := 5
for n := 1 to 5 do
|   y := 10 · y
|   output y
end
output y
```

Output: $\boxed{50\ \ 500\ \ 5000\ \ 50000\ \ 500000;\ 500000}$

9. (10 points) What does the following algorithm do?

```
Algorithm
Input: A list X = (x_1, x_2, ..., x_n) of integers
Output: ?
begin
  sum := 0
  for k := 1 to n do
    if ( x_k = 0 ) then
      sum := sum+1
    end
  end
  output sum
end
```

It steps through the input list, and every time it encounters a zero it increments the variable *sum*. Therefore the algorithm $\boxed{\textbf{counts the number of 0's that occur in the list } X\textbf{.}}$

10. (10 points) Write an algorithm whose input is a positive integer n and whose output is the first n terms of the sequence $6, 16, 26, 36, 46, 56, \ldots$.

This sequence starts with 6, and to get the next term we always add 10.

```
Algorithm
Input: A positive integer n.
Output: First n terms of sequence 6, 16, 26, 36, ...
begin
  a := 6
  for i = 1 to n do
    output a  ................(this is the ith term)
    a := a + 10  ..........(now add 10 to the ith term to get the next term)
  end
end
```