Name: Richard

QUIZ 11 ♠

MATH 211
February 23, 2023

1. Expand and simplify: $(1+a)^5 = 1\cdot 1^5 a^0 + 5\cdot 1^4 a^1 + 10\cdot 1^3 a^2 + 10\cdot 1^2 a^3 + 5\cdot 1^1 a^4 + 1\cdot 1^0 a^5$

$$= \boxed{1 + 5a + 10a^2 + 10a^3 + 5a^4 + a^5}$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

2. Use the binomial theorem to show why $3^n = 2^0\binom{n}{0} + 2^1\binom{n}{1} + 2^2\binom{n}{2} + 2^3\binom{n}{3} + \cdots + 2^n\binom{n}{n}$

$$3^n = (1+2)^n = \binom{n}{0}1^n\cdot 2^0 + \binom{n}{1}1^{n-1}2^1 + \binom{n}{2}1^{n-2}2^2 + \binom{n}{3}1^{n-3}\cdot 2^3 + \cdots + \binom{n}{n}1^0 2^n$$

$$= 2^0\binom{n}{0} + 2^1\binom{n}{1} + 2^2\binom{n}{2} + 2^3\binom{n}{3} + \cdots + 2^n\binom{n}{n}$$

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QUIZ 11 ♢

MATH 211
February 23, 2023

1. Expand and simplify: $(a+2)^4 = 1\cdot a^4 2^0 + 4a^3\cdot 2^1 + 6a^2 2^2 + 4a^1 2^3 + 1\cdot a^0 2^4$

$$= \boxed{a^4 + 8a^3 + 24a^2 + 32a + 16}$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

2. Use the binomial theorem to show why $4^n = 3^0\binom{n}{0} + 3^1\binom{n}{1} + 3^2\binom{n}{2} + 3^3\binom{n}{3} + \cdots + 3^n\binom{n}{n}$

$$4^n = (1+3)^n = \binom{n}{0}1^n\cdot 3^0 + \binom{n}{1}1^{n-1}3^1 + \binom{n}{2}1^{n-2}3^2 + \cdots + \binom{n}{n}1^0 3^n$$

$$= 3^0\binom{n}{0} + 3^1\binom{n}{1} + 3^2\binom{n}{2} + \cdots + 3^n\binom{n}{n}$$

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QUIZ 11 ♣

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February 23, 2023

1. Expand and simplify: $(1+a)^6 = 1\cdot 1^6 a^0 + 6\cdot 1^5 a + 15\cdot 1^4 a^2 + 20\cdot 1^3 a^3 + 15\cdot 1^2 a^4 + 6\cdot 1\cdot a^5 + 1\cdot a^6$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

$$= \boxed{1 + 6a + 15a^2 + 20a^3 + 15a^4 + 6a^5 + a^6}$$

2. Use the binomial theorem to show why $2^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \cdots + \binom{n}{n}$

$$2^n = (1+1)^n = \binom{n}{0}1^n\cdot 1^0 + \binom{n}{1}1^{n-1}1 + \binom{n}{2}1^{n-2}1^2 + \binom{n}{3}1^{n-3}1^3 + \cdots + \binom{n}{n}1^0 1^n$$

$$= \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \cdots + \binom{n}{n}$$

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QUIZ 11 ♡

MATH 211

February 23, 2023

1. Expand and simplify: $(a+2)^4 = 1\cdot a^4 2^0 + 4a^3\cdot 2 + 6a^2\cdot 2^2 + 4a\cdot 2^3 + 1\cdot a^0 2^4$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$$= \boxed{a^4 + 8a^3 + 24a^2 + 32a + 16}$$

2. Use the binomial theorem to show why $3^n = 2^0\binom{n}{0} + 2^1\binom{n}{1} + 2^2\binom{n}{2} + 2^3\binom{n}{3} + \cdots + 2^n\binom{n}{n}$

$$3^n = (1+2)^n = \binom{n}{0}1^n\cdot 2^0 + \binom{n}{1}1^{n-1}2^1 + \binom{n}{2}1^{n-2}2^2 + \binom{n}{3}1^{n-3}2^3 + \cdots + \binom{n}{n}1^0\cdot 2^n$$

$$= 2^0\binom{n}{0} + 2\binom{n}{1} + 2^2\binom{n}{2} + 2^3\binom{n}{3} + \cdots + 2^n\binom{n}{n}$$