Name: Richard

QUIZ 6 ♠

MATH 201
February 6, 2024

1. The curve $y = \frac{1}{3}x^3$ for $1 \le x \le 2$ is rotated around the x-axis.

Find the area of the resulting surface.

$$SA = \int_1^2 2\pi f(x) \sqrt{1 + (f'(x))^2}\, dx$$

$$= \int_1^2 2\pi \frac{1}{3} x^3 \sqrt{1 + (x^2)^2}\, dx$$

$$= \frac{2\pi}{3} \int_1^2 x^3 \sqrt{1 + x^4}\, dx$$

$$u = 1 + x^4$$
$$\frac{du}{dx} = 4x^3$$
$$du = 4x^3 dx$$
$$\frac{1}{4} du = x^3 dx$$

$$= \frac{2}{3}\pi \int_1^2 \sqrt{1 + x^4}\, x^3 dx$$

$$= \frac{2}{3}\pi \int_{1+1^4}^{1+2^4} \sqrt{u}\, \frac{1}{4}\, du$$

$$= \frac{\pi}{6} \int_2^{17} u^{1/2}\, du = \frac{\pi}{6}\left[\frac{u^{3/2}}{3/2}\right]_2^{17}$$

$$= \frac{\pi}{6}\left[\frac{2\sqrt{u}^3}{3}\right]_2^{17} = \frac{\pi}{6}\left(\frac{2\sqrt{17}^3}{3} - \frac{2\sqrt{2}^3}{3}\right)$$

$$= \frac{\pi}{6}\left(\frac{34\sqrt{17}}{3} - \frac{4\sqrt{2}}{3}\right) = \boxed{\frac{\pi}{9}\left(17\sqrt{17} - 2\sqrt{2}\right) \text{ square units}}$$

Name: Richard

QUIZ 6 ◇

MATH 201
February 6, 2024

1. The curve $y = \sqrt{1-x^2}$ for $-1/2 \le x \le 1/2$ is rotated around the x-axis. Find the area of the resulting surface.

$$SA = \int_{-1/2}^{1/2} 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx$$

$$y = (1-x^2)^{\frac{1}{2}}$$

$$y' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)$$

$$= \frac{-2x}{2\sqrt{1-x^2}}$$

$$= \frac{-x}{\sqrt{1-x^2}}$$

$$= 2\pi\int_{-1/2}^{1/2}\sqrt{1-x^2}\sqrt{1+\left(\frac{-x}{\sqrt{1-x^2}}\right)^2}\,dx$$

$$= 2\pi\int_{-1/2}^{1/2}\sqrt{1-x^2}\sqrt{1+\frac{x^2}{1-x^2}}\,dx$$

$$= 2\pi\int_{-1/2}^{1/2}\sqrt{1-x^2}\sqrt{\frac{1-x^2+x^2}{1-x^2}}\,dx$$

$$= 2\pi\int_{-1/2}^{1/2}\sqrt{1-x^2}\,\frac{1}{\sqrt{1-x^2}}\,dx$$

$$= 2\pi\int_{-1/2}^{1/2}dx = 2\pi\Big[x\Big]_{-1/2}^{1/2}$$

$$= 2\pi\left(\frac{1}{2}-\left(-\frac{1}{2}\right)\right) = \boxed{2\pi \text{ square units}}$$

Name: Richard

QUIZ 6 ♣

MATH 201
February 6, 2024

1. The curve $y = 2\sqrt{x}$ for $0 \le x \le 3$ is rotated around the x-axis.
   Find the area of the resulting surface.

$$SA = \int_0^3 2\pi f(x)\sqrt{1 + (f'(x))^2}\, dx$$

$$= \int_0^3 2\pi \, 2\sqrt{x}\sqrt{1 + \left(\frac{2}{2\sqrt{x}}\right)^2}\, dx$$

$$= \int_0^3 4\pi\sqrt{x}\sqrt{1 + \frac{1}{x}}\, dx$$

$$= \int_0^3 4\pi\sqrt{x}\sqrt{\frac{x+1}{x}}\, dx$$

$$= 4\pi\int_0^3 \sqrt{x}\,\frac{\sqrt{x+1}}{\sqrt{x}}\, dx$$

$$u = x + 1 \qquad \frac{du}{dx} = 1 \qquad du = dx$$

$$= 4\pi\int_0^3 \sqrt{x+1}\, dx$$

$$= 4\pi\int_{0+1}^{3+1} \sqrt{u}\, du = 4\pi\int_1^4 u^{1/2}\, du$$

$$= 4\pi\left[\frac{u^{3/2}}{3/2}\right]_1^4 = 4\pi\left[\frac{2\sqrt{u}^3}{3}\right]_1^4 = \frac{8\pi}{3}\left[\sqrt{u}^3\right]_1^4$$

$$= \frac{8\pi}{3}\left(\sqrt{4}^3 - \sqrt{1}^3\right) = \frac{8\pi}{3}(8-1) = \boxed{\frac{56\pi}{3} \text{ square units}}$$

Name: Richard	QUIZ 6 ♡	MATH 201 February 6, 2024

1. The curve $y = \frac{1}{2}\left(e^{x} + e^{-x}\right)$ for $0 \leq x \leq 2$ is rotated around the x-axis.

   Find the area of the resulting surface.

$$SA = \int_0^2 2\pi f(x)\sqrt{1 + (f'(x))^2}\,dx$$

$$= \int_0^2 2\pi \frac{1}{2}(e^x + e^{-x})\sqrt{1 + \left(\frac{1}{2}(e^x - e^{-x})\right)^2}\,dx$$

$$= \pi \int_0^2 (e^x + e^{-x})\sqrt{1 + \frac{1}{4}(e^{2x} - 1 - 1 + e^{-2x})}\,dx$$

$$= \pi \int_0^2 (e^x + e^{-x})\sqrt{\frac{e^{2x}}{4} + \frac{1}{2} + \frac{e^{-2x}}{4}}\,dx$$

$$= \pi \int_0^2 (e^x + e^{-x})\sqrt{\left(\frac{e^x}{2} + \frac{e^{-x}}{2}\right)^2}\,dx$$

$$= \pi \int_0^2 (e^x + e^{-x})\left(\frac{e^x}{2} + \frac{e^{-x}}{2}\right)dx$$

$$= \frac{\pi}{2}\int_0^2 (e^x + e^{-x})(e^x + e^{-x})\,dx$$

$$= \frac{\pi}{2}\int_0^2 \left(e^{2x} + 1 + 1 + e^{-2x}\right)dx$$

$$= \frac{\pi}{2}\left[\frac{e^{2x}}{2} + 2x - \frac{e^{-2x}}{2}\right]_0^2$$

$$= \frac{\pi}{2}\left(\left(\frac{e^4}{2} + 4 - \frac{e^{-4}}{2}\right) - \left(\frac{e^0}{2} + 2\cdot 0 - \frac{e^0}{2}\right)\right)$$

$$= \boxed{\frac{\pi}{4}\left(e^4 - e^{-4} + 8\right) \text{ square units}}$$