

1. (4 pts.) Determine whether the sequence converges or diverges. If it converges, give the value if possible.

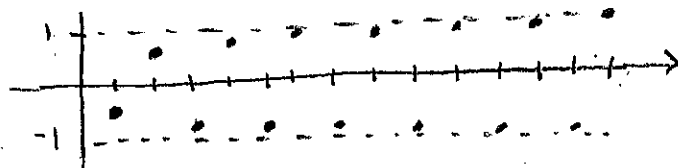
(a) $\left\{ \frac{4 \tan^{-1}(n)}{\pi} \right\}_{n=1}^{\infty}$

$$\lim_{n \rightarrow \infty} \frac{4 \tan^{-1}(n)}{\pi} = \frac{4}{\pi} \lim_{n \rightarrow \infty} \tan^{-1}(n) = \frac{4}{\pi} \cdot \frac{\pi}{2} = 2$$

2
Converges to 2

(b) $\left\{ \frac{(-1)^n(n+1)}{n+2} \right\}_{n=1}^{\infty}$

Diverges



Note that $\lim_{n \rightarrow \infty} \frac{n+1}{n+2} = 1$, but the $(-1)^n$ makes the sequence terms alternate positive and negative getting closer to 1 and -1. Limit DNE.

2. (6 pts.) Determine whether the series converges or diverges. If it converges, give the value if possible.

(a) $\sum_{k=0}^{\infty} \frac{3}{4^k} = \frac{a}{1-r} = \frac{3}{1-\frac{1}{4}} = \frac{3}{\frac{3}{4}} = 4 \leftarrow \text{converges}$

(Geometric series $a=3$, $r=\frac{1}{4}$)

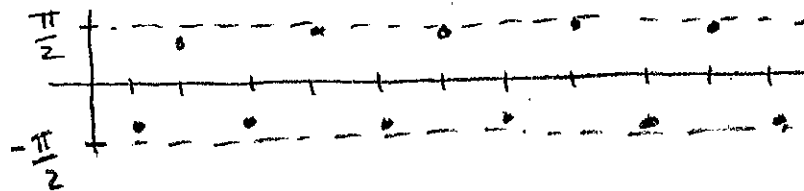
(b) $\sum_{k=1}^{\infty} \left(\frac{5}{k} - \frac{5}{k+1} \right) = \lim_{n \rightarrow \infty} S_n$

$$= \lim_{n \rightarrow \infty} \left(\left(\frac{5}{1} - \frac{5}{2} \right) + \left(\frac{5}{2} - \frac{5}{3} \right) + \left(\frac{5}{3} - \frac{5}{4} \right) + \left(\frac{5}{4} - \frac{5}{5} \right) + \dots + \left(\frac{5}{n} - \frac{5}{n+1} \right) \right)$$

$$= \lim_{n \rightarrow \infty} \left(5 - \frac{5}{n+1} \right) = 5 - 0 = 5 \leftarrow \text{converges}$$

1. (4 pts.) Determine whether the sequence converges or diverges. If it converges, give the value if possible.

(a) $\{(-1)^n \tan^{-1}(n)\}_{n=1}^{\infty}$



Diverges Sequence terms alternate signs, getting progressively closer to $\pi/2$ and $-\pi/2$

(b) $\frac{\ln(2)}{2}, \frac{\ln(3)}{3}, \frac{\ln(4)}{4}, \frac{\ln(5)}{5}, \dots = \left\{ \frac{\ln(n)}{n} \right\}_{n=1}^{\infty}$

$$\lim_{n \rightarrow \infty} \frac{\ln(n)}{n} = \lim_{n \rightarrow \infty} \frac{1/n}{1} = \frac{0}{1} = \boxed{0} \leftarrow \boxed{\text{converges!}}$$

form $\frac{\infty}{\infty}$ L'Hôpital

2. (6 pts.) Determine whether the series converges or diverges. If it converges, give the value if possible.

(a) $\sum_{k=0}^{\infty} \frac{5}{2^k} = \frac{a}{1-r} = \frac{5}{1-1/2} = \frac{5}{1/2} = \boxed{10} \leftarrow \boxed{\text{converges}}$

(Geometric series $a=5$ $r=1/2$)

(b) $\sum_{k=1}^{\infty} \left(\sqrt{\frac{2}{k}} - \sqrt{\frac{2}{k+1}} \right) = \lim_{n \rightarrow \infty} S_n$

$$= \lim_{n \rightarrow \infty} \left(\left(\sqrt{\frac{2}{1}} - \sqrt{\frac{2}{2}} \right) + \left(\sqrt{\frac{2}{2}} - \sqrt{\frac{2}{3}} \right) + \left(\sqrt{\frac{2}{3}} - \sqrt{\frac{2}{4}} \right) + \left(\sqrt{\frac{2}{4}} - \sqrt{\frac{2}{5}} \right) + \dots + \left(\sqrt{\frac{2}{n}} - \sqrt{\frac{2}{n+1}} \right) \right)$$

$$= \lim_{n \rightarrow \infty} \left(\sqrt{2} - \sqrt{\frac{2}{n+1}} \right) = \sqrt{2} - \sqrt{0} = \boxed{\sqrt{2}}$$

Converges

1. (4 pts.) Determine whether the sequence converges or diverges. If it converges, give the value if possible.

$$(a) \left\{ \cos\left(\frac{\pi n}{3n+1}\right) \right\}_{n=1}^{\infty}$$

$$\lim_{n \rightarrow \infty} \cos\left(\frac{\pi n}{3n+1}\right) = \cos\left(\lim_{n \rightarrow \infty} \frac{\pi n}{3n+1}\right) = \cos\left(\frac{\pi}{3}\right) = \boxed{\frac{1}{2}}$$

Converges!

$$(b) \frac{\ln(2)+1}{\ln(2)+2}, \frac{\ln(3)+1}{\ln(3)+2}, \frac{\ln(4)+1}{\ln(4)+2}, \frac{\ln(5)+1}{\ln(5)+2}, \dots = \left\{ \frac{\ln(n)+1}{\ln(n)+2} \right\}_{n=2}^{\infty}$$

$$\lim_{n \rightarrow \infty} \frac{\ln(n)+1}{\ln(n)+2} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n} + 0}{\frac{1}{n} + 0} = \lim_{n \rightarrow \infty} 1 = \boxed{1}$$

Converges!

form $\frac{\infty}{\infty}$

L'Hôpital

2. (6 pts.) Determine whether the series converges or diverges. If it converges, give the value if possible.

$$(a) \sum_{k=0}^{\infty} \frac{7}{5^k} = \frac{a}{1-r} = \frac{7}{1-\frac{1}{5}} = \frac{7}{\frac{4}{5}} = \boxed{\frac{35}{4}}$$

Converges!

(Geometric series, $a=7$, $r=1/5$)

$$(b) \sum_{k=1}^{\infty} (e^{1-k} - e^{-k}) = \lim_{n \rightarrow \infty} S_n$$

$$= \lim_{n \rightarrow \infty} \left(\underbrace{(e^0 - e^{-1})}_{\uparrow} + \underbrace{(e^{-1} - e^{-2})}_{\uparrow} + \underbrace{(e^{-2} - e^{-3})}_{\uparrow} + \underbrace{(e^{-3} - e^{-4})}_{\uparrow} + \dots + \underbrace{(e^{1-n} - e^{-n})}_{\uparrow} \right)$$

$$= \lim_{n \rightarrow \infty} (e^0 - e^{-n}) = \lim_{n \rightarrow \infty} \left(1 - \frac{1}{e^n} \right) = 1 - 0 = \boxed{1}$$

Converges!

1. (4 pts.) Determine whether the sequence converges or diverges. If it converges, give the value if possible.

(a) $\left\{ \frac{\sin(n)}{n} \right\}_{n=1}^{\infty}$

$$\lim_{n \rightarrow \infty} \frac{\sin(n)}{n} = \boxed{0} \leftarrow \boxed{\text{converges!}}$$

(b) $\left\{ \frac{n}{e^n + n} \right\}_{n=1}^{\infty}$

$$\lim_{n \rightarrow \infty} \frac{n}{e^n + 1} = \lim_{n \rightarrow \infty} \frac{1}{e^n + 0} = \lim_{n \rightarrow \infty} \frac{1}{e^n} = \boxed{0} \leftarrow \boxed{\text{converges!}}$$

form $\frac{\infty}{\infty}$
L'Hôpital

2. (6 pts.) Determine whether the series converges or diverges. If it converges, give the value if possible.

(a) $\sum_{k=0}^{\infty} \frac{4}{3^k} = \frac{a}{1-r} = \frac{4}{1-1/3} = \frac{4}{2/3} = \boxed{6} \leftarrow \boxed{\text{Converges!}}$

(b) $\sum_{k=1}^{\infty} (\ln(k) - \ln(k+1)) = \lim_{n \rightarrow \infty} S_n$

$$= \lim_{n \rightarrow \infty} \left((\ln(1) - \ln(2)) + (\ln(2) - \ln(3)) + (\ln(3) - \ln(4)) + \dots + (\ln(n) - \ln(n-1)) \right)$$

$$= \lim_{n \rightarrow \infty} (0 - \ln(n-1)) = \boxed{-\infty} \leftarrow \boxed{\text{Diverges!}}$$