Name: Richard

QUIZ 16 ♠

MATH 201
March 26, 2024

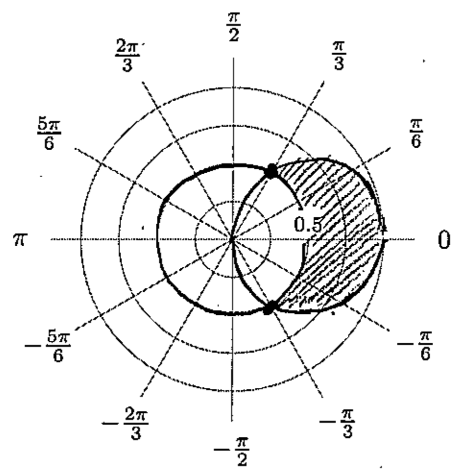
1. Find the area of the region outside the circle $r = \dfrac{1}{2}$ and inside the circle $r = \cos(\theta)$.
(Find intersection points and sketch the curves first.)

Intersection points:

Solve $\frac{1}{2} = \cos\theta$

$\theta = \frac{\pi}{3}, -\frac{\pi}{3}$

Intersections: $\left(\frac{1}{2}, \pi/3\right)$ $\left(\frac{1}{2}, -\pi/3\right)$



$$A = \frac{1}{2}\int_{-\pi/3}^{\pi/3} \cos^2\theta - \left(\frac{1}{2}\right)^2 d\theta$$

$$= \frac{1}{2}\left[\frac{\theta}{2} + \frac{\cos\theta\sin\theta}{2} - \frac{\theta}{4}\right]_{-\pi/3}^{\pi/3}$$

$$= \frac{1}{2}\left[\frac{\cos\theta\sin\theta}{2} + \frac{\theta}{4}\right]_{-\pi/3}^{\pi/3}$$

$$= \frac{1}{2}\left(\left(\frac{\cos\frac{\pi}{3}\sin\frac{\pi}{3}}{2} + \frac{\pi/3}{4}\right) - \left(\frac{\cos -\pi/3 \sin -\pi/3}{2} + \frac{-\pi/3}{4}\right)\right)$$

$$= \frac{1}{2}\left(\frac{\frac{1}{2}\frac{\sqrt{3}}{2}}{2} + \frac{\pi}{12} - \frac{\frac{1}{2}\left(\frac{-\sqrt{3}}{2}\right)}{2} + \frac{\pi}{12}\right)$$

$$= \frac{1}{2}\left(\frac{\sqrt{3}}{8} + \frac{\pi}{12} + \frac{\sqrt{3}}{8} + \frac{\pi}{12}\right) = \frac{1}{2}\left(\frac{\sqrt{3}}{4} + \frac{\pi}{6}\right)$$

$$= \boxed{\frac{\sqrt{3}}{8} + \frac{\pi}{12} \text{ square units}}$$

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QUIZ 16 ◇

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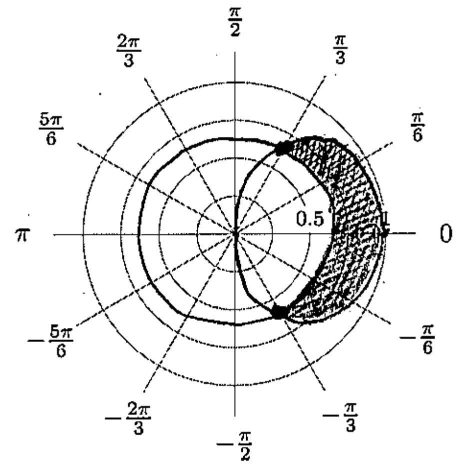
1. Find the area of the region inside the curve $r = \sqrt{\cos(\theta)}$ and outside the circle $r = \frac{1}{\sqrt{2}}$.

(Find intersection points and sketch the curves first. Note: $\frac{1}{\sqrt{2}} \approx 0.7$)



Intersection points:

Solve $\sqrt{\cos\theta} = \frac{1}{\sqrt{2}}$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \pm\frac{\sqrt{3}}{2}$$

Intersections: $\left(\frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}\right), \left(\frac{1}{\sqrt{2}}, -\frac{\sqrt{3}}{2}\right)$

$$A = \frac{1}{2}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sqrt{\cos\theta}^{\,2}\, d\theta - \left(\frac{1}{\sqrt{2}}\right)^2 d\theta$$

$$= \frac{1}{2}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos\theta\, d\theta - \frac{1}{2}\, d\theta$$

$$= \frac{1}{2}\left[\sin\theta - \frac{\theta}{2}\right]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$$

$$= \frac{1}{2}\left(\left(\sin\frac{\pi}{3} - \frac{\pi/3}{2}\right) - \left(\sin -\frac{\pi}{3} - \frac{-\pi/3}{2}\right)\right)$$

$$= \frac{1}{2}\left(\frac{\sqrt{3}}{2} - \frac{\pi}{6} + \frac{\sqrt{3}}{2} - \frac{\pi}{6}\right) = \frac{1}{2}\left(\sqrt{3} - \frac{\pi}{3}\right)$$

$$= \boxed{\frac{\sqrt{3}}{2} - \frac{\pi}{6} \text{ square units}}$$

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Quiz 16 ♣

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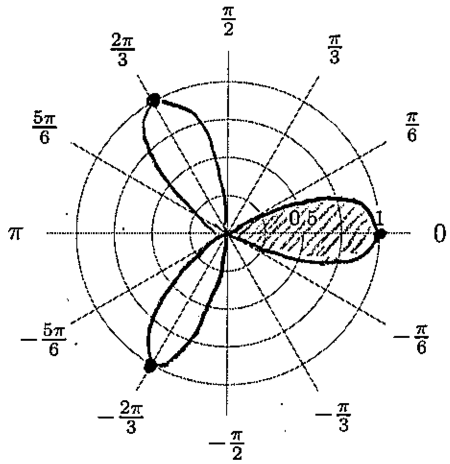
1. Find the area inside one leaf of the rose $r = \cos(3\theta)$.
(Sketch the curve first.)

$$A = \frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \cos^2(3\theta)\,d\theta$$

$u = 3\theta$
$du = 3\,d\theta$
$\frac{1}{3}du = d\theta$



$$= \frac{1}{2}\int_{3(-\frac{\pi}{6})}^{3\cdot\frac{\pi}{6}} \cos^2(u)\,\frac{1}{3}\,du$$

$$= \frac{1}{6}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2(u)\,du$$

$$= \frac{1}{6}\left[\frac{u}{2} + \frac{\cos(u)\sin(u)}{2}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{6}\left(\left(\frac{\pi/2}{2} + \frac{\cos(\pi/2)\sin(\pi/2)}{2}\right) - \left(\frac{-\pi/2}{2} + \frac{\cos(-\pi/2)\sin(-\pi/2)}{2}\right)\right)$$

$$= \frac{1}{6}\left(\left(\frac{\pi}{4} + \frac{0\cdot 1}{2}\right) - \left(-\frac{\pi}{4} + \frac{0\cdot(-1)}{2}\right)\right)$$

$$= \frac{1}{6}\left(\frac{\pi}{4} + \frac{\pi}{4}\right) = \frac{1}{6}\,\frac{\pi}{2} = \boxed{\frac{\pi}{12}\text{ square units}}$$

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QUIZ 16 ♡

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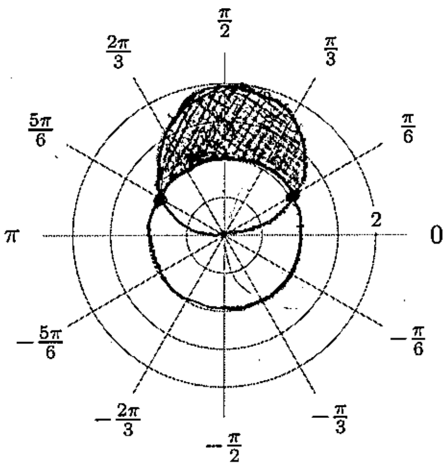
1. Find the area contained between the circles $r = 1$ and $r = 2\sin(\theta)$.
(Find intersection points and sketch the curves first.)

To find intersections, solve

$$1 = 2\sin\theta$$

$$\frac{1}{2} = \sin\theta$$

$$\theta = \frac{\pi}{6}, \ \frac{5\pi}{6}$$



Intersections: $\left(1, \frac{\pi}{6}\right), \left(1, \frac{5\pi}{6}\right)$

$$A = \frac{1}{2}\int_{\pi/6}^{5\pi/6} (2\sin(\theta))^2 - 1^2 \, d\theta$$

$$= \frac{1}{2}\int_{\pi/6}^{5\pi/6} 4\sin^2(\theta) - 1 \, d\theta$$

$$= \frac{1}{2}\left[4\left(\frac{\theta}{2} - \frac{\sin(\theta)\cos(\theta)}{2}\right) - \theta\right]_{\pi/6}^{5\pi/6}$$

$$= \frac{1}{2}\Big[\theta - 2\sin(\theta)\cos(\theta)\Big]_{\pi/6}^{5\pi/6}$$

$$= \frac{1}{2}\left(\left(\frac{5\pi}{6} - 2\sin\left(\frac{5\pi}{6}\right)\cos\left(\frac{5\pi}{6}\right)\right) - \left(\frac{\pi}{6} - 2\sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}\right)\right)\right)$$

$$= \frac{1}{2}\left(\frac{5\pi}{6} - 2\,\frac{1}{2}\left(-\frac{\sqrt{3}}{2}\right) - \frac{\pi}{6} + 2\,\frac{1}{2}\,\frac{\sqrt{3}}{2}\right) = \frac{1}{2}\left(\frac{4\pi}{6} + \sqrt{3}\right)$$

$$= \boxed{\frac{\pi}{3} + \frac{\sqrt{3}}{2} \text{ square units}}$$