Name: Richard

QUIZ 11 ♠

MATH 201

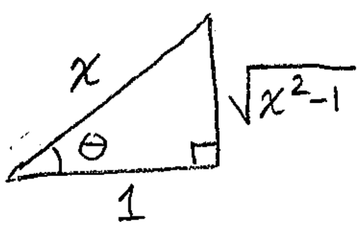
February 27, 2024

1. $$\int \frac{1}{\sqrt{x^2-1}}\,dx = \int \frac{1}{\sqrt{\sec^2(\theta)-1}} \sec(\theta)\tan(\theta)\,d\theta$$

$x = \sec(\theta)$

$dx = \sec(\theta)\tan(\theta)\,d\theta$

$\sec(\theta) = \frac{x}{1} = \frac{HYP}{ADJ}$



$$= \int \frac{1}{\sqrt{\tan^2(\theta)}} \sec(\theta)\tan(\theta)\,d\theta$$

$$= \int \frac{1}{\tan(\theta)} \sec(\theta)\tan(\theta)\,d\theta$$

$$= \int \sec(\theta)\,d\theta$$

$$= \ln\left|\sec(\theta) + \tan(\theta)\right| + C$$

$$= \ln\left|\frac{HYP}{ADJ} + \frac{OPP}{ADJ}\right| + C$$

$$= \ln\left|\frac{x}{1} + \frac{\sqrt{x^2-1}}{1}\right| + C$$

$$= \boxed{\ln\left|x + \sqrt{x^2-1}\right| + C}$$

Check $$\frac{d}{dx}\left[\ln\left|x+\sqrt{x^2-1}\right| + C\right] = \frac{1 + \frac{2x}{2\sqrt{x^2-1}}}{x + \sqrt{x^2+1}}$$

$$= \frac{1 + \frac{x}{\sqrt{x^2-1}}}{x+\sqrt{x^2+1}} = \frac{\frac{\sqrt{x^2-1} + x}{\sqrt{x^2-1}}}{\frac{x+\sqrt{x^2-1}}{1}} = \frac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}} \cdot \frac{1}{x+\sqrt{x^2-1}} = \boxed{\frac{1}{\sqrt{x^2-1}}}$$ ✓

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Quiz 11 ◊

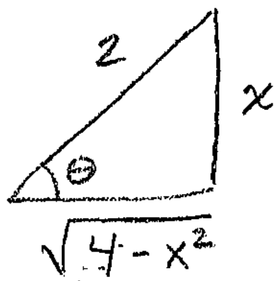
MATH 201
February 27, 2024

1. $\int \frac{1}{x^2\sqrt{4-x^2}}\,dx = \int \frac{1}{(2\sin(\theta))^2\sqrt{4-(2\sin(\theta))^2}}\,2\cos(\theta)\,d\theta$

$x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

$\sin(\theta) = \frac{x}{2} = \frac{OPP}{HYP}$



$= \int \frac{1}{4\sin^2(\theta)\sqrt{4-4\sin^2(\theta)}}\,2\cos(\theta)\,d\theta$

$= \int \frac{2\cos(\theta)}{4\sin^2(\theta)\,2\sqrt{1-\sin^2(\theta)}}\,d\theta$

$= \int \frac{2\cos(\theta)}{8\sin^2(\theta)\sqrt{\cos^2(\theta)}}\,d\theta$

$= \int \frac{\cos(\theta)}{4\sin^2(\theta)\cos(\theta)}\,d\theta = \frac{1}{4}\int \frac{d\theta}{\sin^2(\theta)} = \frac{1}{4}\int \csc^2(\theta)\,d\theta$

$= -\frac{1}{4}\cot(\theta) + C = -\frac{1}{4}\frac{ADJ}{OPP} + C = \boxed{-\frac{\sqrt{4-x^2}}{4x} + C}$

Check $\frac{d}{dx}\left[-\frac{\sqrt{4-x^2}}{4x}\right] = \frac{-1}{4}\,\frac{\frac{-2x}{2\sqrt{4-x^2}}x - 1\cdot\sqrt{4-x^2}}{x^2} = \frac{\frac{x^2}{\sqrt{4-x^2}} + \sqrt{4-x^2}}{4x^2}$

$= \frac{\frac{x^2}{\sqrt{4-x^2}} + \frac{4-x^2}{\sqrt{4-x^2}}}{\frac{4x^2}{1}} = \frac{\frac{4}{\sqrt{4-x^2}}}{\frac{4x^2}{1}} = \frac{4}{\sqrt{4-x^2}}\,\frac{1}{4x^2} = \frac{1}{x^2\sqrt{4-x^2}}$

✓

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Quiz 11 ♣

MATH 201

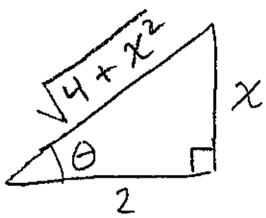
February 27, 2024

1. $\int \frac{1}{\sqrt{4+x^2}}\,dx = \int \frac{1}{\sqrt{4+(2\tan(\theta))^2}}\, 2\sec^2(\theta)\,d\theta$

$x = 2\tan(\theta)$

$dx = 2\sec^2(\theta)\,d\theta$

$\tan(\theta) = \frac{x}{2} = \frac{OPP}{ADJ}$



$= \int \frac{2\sec^2(\theta)}{\sqrt{4+4\tan^2(\theta)}}\,d\theta$

$= \int \frac{2\sec^2(\theta)}{2\sqrt{1+\tan^2\theta}}\,d\theta$

$= \int \frac{\sec^2(\theta)}{\sqrt{\sec^2(\theta)}}\,d\theta = \int \sec(\theta)\,d\theta$

$= \ln\left|\sec(\theta) + \tan(\theta)\right| + C$

$= \ln\left|\frac{HYP}{ADJ} + \frac{OPP}{ADJ}\right| + C$

$= \ln\left|\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}\right| + C = \ln\left|\frac{1}{2}\left(\sqrt{4+x^2} + x\right)\right| + C$

$= \ln\left(\frac{1}{2}\right) + \ln\left|\sqrt{4+x^2} + x\right| + C = \boxed{\ln\left|\sqrt{4+x^2} + x\right| + C}$

Check: $\frac{d}{dx}\left[\ln\left|\sqrt{4+x^2} + x\right| + C\right] = \frac{\frac{2x}{2\sqrt{4+x^2}} + 1}{\sqrt{4+x^2} + x}$

$= \frac{\frac{x+\sqrt{4+x^2}}{\sqrt{4+x^2}}}{\frac{\sqrt{4+x^2}+x}{1}} = \frac{x+\sqrt{4+x^2}}{\sqrt{4+x^2}} \cdot \frac{1}{\sqrt{4+x^2}+x} = \frac{1}{\sqrt{4+x^2}}$ ✓

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Quiz 11 ♡

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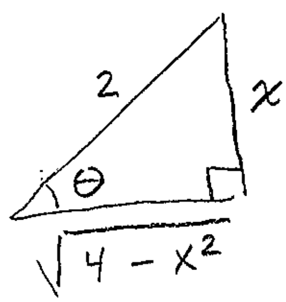
February 27, 2024

1. $\int \sqrt{4-x^2}\,dx = \int \sqrt{4-(2\sin(\theta))^2}\; 2\cos(\theta)\,d\theta$

$x = 2\sin(\theta)$

$dx = 2\cos(\theta)\,d\theta$

$\sin(\theta) = \frac{x}{2} = \frac{\text{OPP}}{\text{HYP}}$



$\theta = \sin^{-1}\left(\frac{x}{2}\right)$

$= \int \sqrt{4-4\sin^2(\theta)}\; 2\cos(\theta)\,d\theta$

$= \int 2\sqrt{1-\sin^2(\theta)}\; 2\cos(\theta)\,d\theta$

$= 4\int \sqrt{\cos^2(\theta)}\;\cos(\theta)\,d\theta$

$= 4\int \cos^2(\theta)\,d\theta$

$= 4\left(\frac{\theta}{2} + \frac{\cos(\theta)\sin(\theta)}{2}\right)$

$= 2\theta + 2\cos(\theta)\sin(\theta) + C$

$= 2\sin^{-1}\left(\frac{x}{2}\right) + 2\,\frac{\text{ADJ}}{\text{HYP}}\cdot\frac{\text{OPP}}{\text{HYP}} + C$

$= 2\sin^{-1}\left(\frac{x}{2}\right) + 2\,\frac{\sqrt{4-x^2}}{2}\,\frac{x}{2} + C$

$= \boxed{2\sin^{-1}\left(\frac{x}{2}\right) + \frac{1}{2}x\sqrt{4-x^2} + C}$