

$$1. \int \tan^3(x) \sec^8(x) dx = \int \tan^2(x) \sec^7(x) \sec(x) \tan(x) dx$$

$$\begin{array}{l} u = \sec(x) \\ du = \sec(x) \tan(x) dx \end{array} = \int (\sec^2(x) - 1) \sec^7(x) \sec(x) \tan(x) dx$$

$$= \int (u^2 - 1) u^7 du$$

$$= \int u^9 - u^7 du$$

$$= \frac{u^{10}}{10} - \frac{u^8}{8} + C$$

$$= \boxed{\frac{\sec^{10}(x)}{10} - \frac{\sec^8(x)}{8} + C}$$

$$1. \int \cos^3(x) \sin^6(x) dx = \int \cos^2(x) \sin^6(x) \cos(x) dx$$

$$\begin{array}{l} u = \sin(x) \\ du = \cos(x) dx \end{array}$$

$$= \int (1 - \sin^2(x)) \sin^6(x) \cos(x) dx$$

$$= \int (1 - u^2) u^6 du$$

$$= \int u^6 - u^8 du = \frac{u^7}{7} - \frac{u^9}{9} + C$$

$$= \boxed{\frac{\sin^7(x)}{7} - \frac{\sin^9(x)}{9} + C}$$

$$\begin{aligned} 1. \int \cos^5(x) dx &= \int \cos^4(x) \cos(x) dx \\ &= \int (\cos^2(x))^2 \cos(x) dx \\ &= \int (1 - \sin^2(x))^2 \cos(x) dx \\ &= \int (1 - u^2)^2 du \\ &= \int 1 - 2u^2 + u^4 du = u - \frac{2u^3}{3} + \frac{u^4}{4} + C \\ &= \boxed{\sin(x) - \frac{2\sin^3(x)}{3} + \frac{\sin^4(x)}{4} + C} \end{aligned}$$

u = sin(x)
du = cos(x) dx

$$\begin{aligned} 1. \int \tan^6(x) \sec^4(x) dx &= \int \tan^6(x) \sec^2(x) \sec^2(x) dx \\ &= \int \tan^6(x) (1 + \tan^2(x)) \sec^2(x) dx \\ &= \int u^6 (1 + u^2) du \\ &= \int u^6 + u^8 du = \frac{u^7}{7} + \frac{u^9}{9} + C \\ &= \boxed{\frac{\tan^7(x)}{7} + \frac{\tan^9(x)}{9} + C} \end{aligned}$$

u = tan(x)
du = sec²(x) dx