Name: _____   SAMPLE MIDTERM   MATH 200, SECTIONS 7 & 8
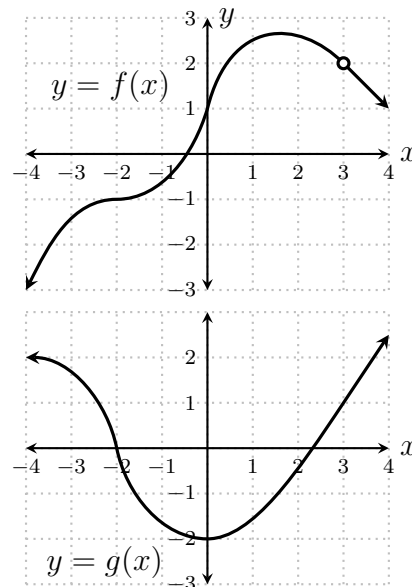
Directions: Closed book, closed notes, no calculators.   Put all phones, etc., away.   You will need only a pencil or pen.

1. (6 points) Answer the questions about the functions graphed below.

   (a) $\lim_{x \to 3} f(x) =$

   (b) $\lim_{x \to 0} \dfrac{2f(x)g(x) + 4f(x)}{g(x) + 2} =$

   (c) $\lim_{x \to 3} g\left(x^2 - 6\right) =$

2. (8 points) Draw the graph of **one** function $f(x)$ meeting **all** of the following conditions.

   (a) The domain of f is $(-\infty, 1) \cup (1, \infty)$.

   (b) The function f is continuous at all x except $x = -2$, $x = 1$ and $x = 4$.

   (c) $\lim_{x \to 1} f(x) = -\infty$
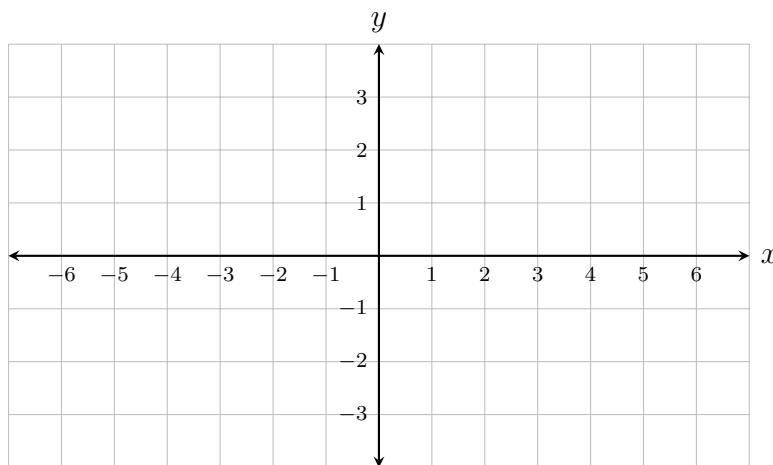
   (d) $\lim_{x \to -2} f(x) = 3$

   (e) $\lim_{x \to 4^-} f(x) = 2$

   (f) $\lim_{x \to 4^+} f(x) = 0$

   (g) $\lim_{x \to \infty} f(x) = 1$

   (h) $\lim_{x \to -\infty} f(x) = 2$

3. (6 points) Find the limits

   (a) $\lim_{x \to 5} \cos\left(\dfrac{\pi x}{3}\right) =$

   (b) $\lim_{x \to 0} \ln\left(4x + e^{x+7}\right) =$
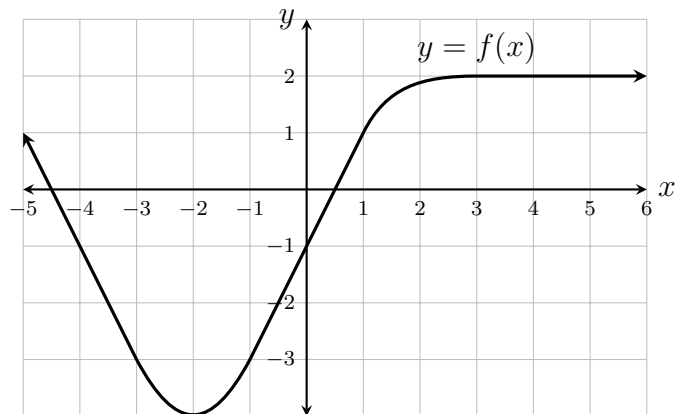
   (c) $\lim_{x \to \infty} \dfrac{4x^3 - 3x + 10}{5x^2 - 6x^3} =$

4. (8 points) Use a **limit definition** of the derivative to find the derivative of $f(x) = \sqrt{x+1}$.

5. (8 points) The graph of a function $f(x)$ is sketched below.

   (a) Using the same coordinate axis, sketch a graph of the derivative $f'(x)$.

   (b) Suppose $g(x) = \dfrac{1}{f(x)}$. Find $g'(0)$.

6. (8 points) Find all x for which the tangent to the graph of $f(x) = x^2 e^x - 2$ at $\big(x, f(x)\big)$ is horizontal.

7. (32 points) Find the derivatives of these functions. You do **not** need to simplify your answers.

(a) $f(x) = 5x^7 + 3x - \sqrt{2}$

(b) $f(x) = \sin(x)\sec(x)$

(c) $f(x) = \sin\left(\sec(x)\right)$

(d) $f(x) = \sin^{-1}\left(x^3 + 3x\right)$

(e) $f(x) = x + \dfrac{\ln(x)}{x}$

(f) $f(x) = \dfrac{1}{\sqrt{e^x + x}}$

(g) $y = \cos\left(e^{x^2 + x}\right)$

(h) Given that $z = w\cos(w)$, find $\dfrac{d^2 z}{dw^2}$.

8. (8 points) A rocket has a height of $t+t^2$ meters t seconds after it is launched. How high is the rocket when its velocity is 101 meters per second?

9. (8 points) Given the equation $\ln|x+y| = xy+1$, find y'.

10. (8 points) A spherical balloon is deflating in such a way that its volume is decreasing at a rate of 18 cubic feet per hour. At what rate is the radius changing when the radius is 3 feet?

Sphere formulas   Volume: $V = \frac{4}{3}\pi r^3$ Surface area: $S = 4\pi r^2$