

1. (35 pts.) Evaluate the following limits. Show steps, as appropriate.

$$(a) \lim_{x \rightarrow 0} \frac{\pi \sin(x)}{3x} =$$

$$(b) \lim_{x \rightarrow \infty} \frac{\sin(x)}{x} =$$

$$(c) \lim_{x \rightarrow \pi/3} \frac{\sin(x)}{x} =$$

$$(d) \lim_{x \rightarrow -\infty} \frac{x^2 - 3x - 10}{x^2 - 8x + 15} =$$

$$(e) \lim_{x \rightarrow 5} \frac{x^2 - 3x - 10}{x^2 - 8x + 15} =$$

$$(f) \lim_{x \rightarrow 0} \frac{(x - 3) \sin(x)}{2x^2 - 6x} =$$

$$(g) \lim_{h \rightarrow 0} \frac{\sqrt{6+h} - \sqrt{6}}{h} =$$

2. (5 pts) Sketch the graph of **one** function with domain $(-\infty, \infty)$ that meets all the following criteria.

(a) $\lim_{x \rightarrow \infty} f(x) = 1$

(b) $\lim_{x \rightarrow -\infty} f(x) = 2$

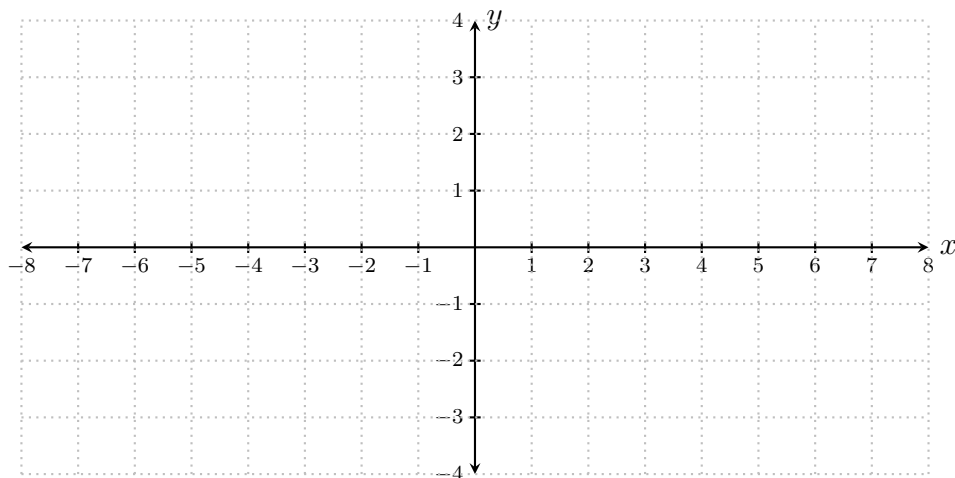
(c) $\lim_{x \rightarrow 4} f(x) = \infty$

(d) $\lim_{x \rightarrow 1^+} f(x) = 1$

(e) $\lim_{x \rightarrow 1^-} f(x) = 2$

(f) $\lim_{x \rightarrow -4} f(x) = 3$

(g) f is not continuous at $x = -4$.



3. (5 pts.) Find the following limit. Explain your reasoning.

$$\lim_{z \rightarrow 5} \frac{\ln(z) - \ln(5)}{z - 5} =$$

4. (5 pts.) Suppose $f(x) = \frac{6}{x}$. Use a limit definition of the derivative to find $f'(x)$.

5. (30 points) Find the derivatives.

$$(a) \frac{d}{dx} [\sin^{-1}(x)] =$$

$$(b) \frac{d}{dx} [\sqrt{x^4 + x^2 + 1}] =$$

$$(c) \frac{d}{dx} [x^2 \cos(x^2)] =$$

$$(d) \frac{d}{dx} \left[\frac{e^x}{x} \right] =$$

$$(e) \frac{d}{dx} \left[\frac{1}{\sqrt{3x+1}} \right] =$$

$$(f) \frac{d}{dx} [\ln(\sec(e^x))] =$$

6. (5 pts.) Consider the equation $x^5 + 4xy^3 - 3y^5 = 2$. Use implicit differentiation to find $\frac{dy}{dx}$.

7. (5 pts.) Use logarithmic differentiation to find the derivative of $f(x) = \left(\frac{1}{x}\right)^x$.

8. (10 pts.) An object is propelled straight down from atop a 160-foot-high tower at time $t = 0$ seconds. At time t seconds its height is $s(t) = 160 - 32t - 16t^2$ feet.

(a) Find the object's height when its velocity is -96 feet per second.

(b) What is object's acceleration when its velocity is -96 feet per second.

9. (**Bonus:** 5 pts.) A plane is taxiing down a runway that is one mile from a tower, as shown below. When the plane is $5/3$ miles from the tower, the distance y between tower and plane is increasing at a rate of 100 mph. What is the plane's velocity at this point in time?

