1. (10 pts.) The graph $y = f'(x)$ of the derivative of a function $f(x)$ is shown. Answer the questions about $f(x)$.

(a) State the intervals on which the function $f(x)$ increases. $(-4, -2)$, $(-2, \infty)$ (where $f'(x) > 0$)
(b) State the intervals on which the function $f(x)$ decreases. $(-\infty, -4)$, $(-2, 0)$ (where $f'(x) < 0$)
(c) State the intervals on which the function $f(x)$ is concave up. $(-\infty, -3)$, $(0, 2)$ (where $f''(x)$ increases)
(d) State the intervals on which the function $f(x)$ is concave down. $(-3, 0)$, $(2, \infty)$ (where $f''(x)$ decreases)
(e) Suppose $f(0) = 0$. Using the above information (and coordinate axis), sketch the graph of $f(x)$.

2. (15 pts.) Find and identify all relative extrema of the function $f(x) = 2 - 3x^4 - 8x^3 - 6x^2$ on the interval $\mathbb{R} = (-\infty, \infty)$. State the extrema in the coordinate form $(x, y)$.

First Derivative Test says

Local max at
$(0, f(0)) = (0, 2)$
No local min

Critical points are $0, -1$

$-1$ $0$

$++ + ++ -- -- -- f'(x)$
3. (15 pts.) US Postal Service regulations state that the length plus girth of a package cannot exceed 108 inches. You must mail a package whose width and height are equal, and with the greatest possible volume. Find the dimensions of the package.

\[ \text{height} = \text{width} = x, \quad \text{length} = y. \]

\[ \begin{align*}
\text{length} + \text{girth} &= 108 \\
4x + 2y &= 108 \\
y &= 108 - 4x
\end{align*} \]

\[ 0 \leq x \leq 27 \]

Volume = \( xy = x^2 y = x^2 (108-4x) \)

\[ \begin{align*}
\text{Volume} &= V(x) = 108x^2 - 4x^3 \\
V'(x) &= 216x - 12x^2 = 12x(18-x) = 0
\end{align*} \]

Critical points: \( x = 0, 18 \)

\[ \begin{align*}
V(x) &= \begin{cases} 
0 & \text{if } x = 0 \\
\text{Max volume when } x = 18 & \text{if } x = 18 \\
36 & \text{if } x = 18
\end{cases}
\end{align*} \]

Answer:

length = \( 36'' \)

width = height = \( 18'' \)

4. (20 points) Evaluate the following limits.

(a) \( \lim_{x \to \pi} \frac{1 + \cos x}{(\pi - x)^2} = \lim_{x \to \pi} \frac{-\sin x}{2(\pi - x)(\pi - x)} = \lim_{x \to \pi} \frac{-\cos x}{2} = -\frac{\cos \pi}{2} = -\frac{-1}{2} = \frac{1}{2} \)

(b) \( \lim_{x \to \infty} x e^{-x} = \lim_{x \to \infty} \frac{x}{e^x} = \lim_{x \to \infty} \frac{1}{e^x} = \frac{1}{\infty} = 0 \)
5. (24 points) Find the indicated indefinite integrals.

(a) \[ \int \left(7 + 7x + \sqrt[3]{x^2} \right) \, dx = \int \left(7 + 7x + x^{2/3} \right) \, dx = 7x + \frac{7}{2} x^2 + \frac{x^{7/5}}{\frac{7}{5}} + C \]

(b) \[ \int \left(e^{4x} + 4 \cos x + 20 \right) \, dx = \frac{1}{4} e^{4x} + 4 \sin x + 20x + C \]

(c) \[ \int \frac{2x}{x^2} \, dx = \int \frac{2}{x} \, dx = 2 \int \frac{1}{x} \, dx = 2 \ln |x| + C \]

6. (8 pts.) Is the equation \( \int (1 + \ln x) \, dx = x + \ln x + C \) true or false? Justify your answer.

Check: \[ \frac{d}{dx} \left[ x + \ln x + C \right] = 1 + \frac{1}{x} + 0 \neq 1 + \ln x. \]

False \( x + \ln x + C \) is not an antiderivative of \( 1 + \ln x \).

7. (8 pts.) Suppose \( f(x) \) is a function for which \( f'(x) = -\sin(x) \) and \( f(2\pi/3) = -3 \). Find \( f(x) \).

\[ f(x) = \int -\sin(x) \, dx = \cos(x) + C \]

\[ f(x) = \cos(x) + C \quad \text{left need to find } C \]

\[ f\left(\frac{2\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right) + C \]

\[ -3 = -\frac{1}{2} + C \]

\[ C = -3 + \frac{1}{2} = -\frac{5}{2} \]

\[ f(x) = \cos(x) - \frac{5}{2} \]